The Lifetime of our Sun

As soon as astronomers realized that the Sun is powered by nuclear fusion reactions, they were in a position to estimate how long it could shine at its current rate of power. We can easily measure the Brightness of the Sun as seen from Earth, at an average distance of 93 Million miles, which I will approximate as 150 million kilometers = 1.5 x 10^8 km = 1.5 x 10^8 km x 1000 meters/km = 1.5 x 10^11 meters. The Sun's brightness here is about one Kilowatt per square meter on a clear day (that is the maximum power you could collect from a perfectly efficient solar cell if they could be made). Since the definition of Brightness is power per surface area:

\[ B = \text{Power} / 4 \pi D^2, \text{ or } P = B \times 4 \pi D^2 = 1000 \text{ Watt/meter}^2 \times 12.5 \times (1.5 \times 10^{11})^2 \]
\[ = 10^3 \times 12.5 \times 2.25 \times 10^{22} = \text{about } 4 \times 10^{26} \text{ Watts} \]

Let's make the approximation (which is not quite right, but is not too far off) that the Sun puts out exactly this amount of power (energy emitted per unit time) over its entire lifetime. How long at that rate does it take the Sun's center to run out of all its nuclear fuel? How much time do we Earthlings have left?? Should we forget about our futures, sell everything, and spend the remaining time partying?? OK, don't answer that last question, it won't be on the exam. Let's get back to the calculation.

We will assume that the Sun started out as 90% Hydrogen atoms. When 4 of these combine in a series of nuclear fusion reactions, we end up with one Helium atom which has somewhat less total mass, by 0.7%. So that means that slightly less than one percent of the fuel was "destroyed"--actually CONVERTED into pure energy using the universally familiar (but not universally understood) formula from Einstein's Special Relativity:

\[ \text{Energy} = 0.007 \times (\text{Mass} \times \text{Speed of Light}^2) \]
\[ = 7 \times 10^{-3} \times 0.90 \times \text{Mass of fuel} \times (3 \times 10^8 \text{ meters/second})^2 \]

Now we must account for the fact that it does not get hot enough for these nuclear fusion reactions to occur except in the Sun's center. So even though the TOTAL mass of the Sun is 2 x 10^{30} kilograms, only the central one tenth of this is available to fuel its enormous power. This makes the total nuclear energy available =

\[ E = 7 \times 10^{-3} \times 0.90 \times 0.10 \times 2 \times 10^{30} \times 9 \times 10^{16} = 1.2 \times 10^{44} \text{ Watt-seconds} \]

To find out its lifetime, just take the ratio of that huge energy to the RATE at which it loses energy:

\[ \text{Time} = \frac{\text{Energy}}{\text{Power}} = \frac{1.2 \times 10^{44} \text{ Watt-seconds}}{4 \times 10^{26} \text{ Watts}} \]
\[ = 3 \times 10^{17} \text{ seconds} = 3 \times 10^{17} / 3 \times 10^7 \text{ seconds/year} = \text{approximately } 10 \text{ Billion years} \]

What a relief! Since the Sun has only been burning fuel for a little under 5 Billion years, it still has somewhat more than half its normal life ahead of it, like some of your parents (or instructors), assuming all goes well. This gives humans A LOT OF TIME to worry about how we will handle things at the end of the Sun's life, when Earth becomes virtually uninhabitable.