Instructions:

1. First read all the questions, then commence with the questions you find easiest.
2. There are 6 questions.
3. Each question carries a weight of 10 points to a total of 60 points.
4. Good Luck!
1. For $s \in [0, 1]$ and $a > 1$ draw the phase portrait of
\[ \dot{x} = s (1 - x) t^a - (1 - s) x (1 - x)^a, \quad x \in [0, 1]. \]
It has three fixed points $0, x^*, 1$ with two being stable and one unstable.
2. Draw the phase portrait of \( \dot{x} = x^2 (6 - x) \) and explain what happens to solutions as \( t \to \infty \) when \( x(0) < 0 \) and \( x(0) > 0 \).
3. Consider $\dot{x} = r + x - x^3$. Draw the bifurcation diagram including types of bifurcations and stability of the fixed points.
4. Consider \( \dot{\theta} = \mu \sin \theta + \sin^2 \theta \). Draw the bifurcation diagram including types of bifurcations and stability of the fixed points.
5. Consider $\dot{x} = rx + \frac{x}{1+x}$. Draw the bifurcation diagram including types of bifurcations and stability of the fixed points.
6. Consider \( \dot{x} = rx - \frac{x^{3}}{1+x} \). Draw the bifurcation diagram including types of bifurcations and stability of the fixed points.