1. Erica claims that this is a graph of the function \[ z = 4e^{-(x^2+y^2)} \]

Draw contour curves for values for \( c=1,2,3,4 \) to see if she is correct. Explain. Label each curve with the correct \( C \) value.

- All level curves are circles.
- Explanations may vary.

\[ C = 1 - \ln\left(\frac{1}{4}\right) \approx 1.39 \quad r \approx 1.18 \]
\[ C = 2 - \ln\left(\frac{1}{2}\right) \approx 0.69 \quad r \approx 0.53 \]
\[ C = 3 - \ln\left(\frac{3}{4}\right) \approx 0.29 \]
\[ C = 4 - \ln(1) = 0 \quad r = 0 \]

The "top" \( C = 4 \) is a point \((0,0)\) at level \( 4 \) as you decrease \( C \), the radii get larger.

\( \checkmark \)
2. Sketch the lines in this system of equations:

\[2x + y = 10\]
\[-3x + 2y = 6\]

\[y = -2x + 10\]
\[y = \frac{3}{2}x + 3\]

Use matrix theory to solve this system and show on your graph what your solution represents.

\[
\begin{pmatrix}
2 & 1 \\
-3 & 2
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
= 
\begin{pmatrix}
10 \\
6
\end{pmatrix}
\]

\[
\begin{pmatrix}
x \\
y
\end{pmatrix}
= \frac{1}{7}
\begin{pmatrix}
2 & -1 \\
3 & 2
\end{pmatrix}
\begin{pmatrix}
10 \\
6
\end{pmatrix}
= \frac{1}{7}
\begin{pmatrix}
14 \\
42
\end{pmatrix}
\]

\[
= \begin{pmatrix}
2 \\
6
\end{pmatrix}
\]

The lines intersect at \(x = 2, \ y = 6\)
3. Suppose we are interested in the population of a certain type of bird in a forest area. We can divide the female population into 2 groups: Hatchlings (< 1 year) and Adults.

Data collected and analyzed has shown that each adult female has on average 2 offspring per year. The survivorship for these adults is $\frac{2}{3}$. (Two-thirds of the adults will survive to breed the next year). The survivorship of the hatchlings is $\frac{1}{3}$. In this patch of forest, there are currently 30 hatchlings and 15 adults. Construct a matrix difference equation for this information and use eigenvalue theory to predict the long-term growth rate of this population and to predict the long-term ratio of hatchlings to the total bird population.

$$
\begin{align*}
X_{n+1} \text{ hatchling (no. of)} &= 2Y_n \\
Y_{n+1} \text{ adults} &= \frac{2}{3}X_n - \frac{1}{2}Y_n
\end{align*}
$$

$$
\begin{pmatrix}
0 & 2 \\
\frac{2}{3} & \frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
X_n \\
Y_n
\end{pmatrix} = 
\begin{pmatrix}
X_{n+1} \\
Y_{n+1}
\end{pmatrix}
$$

Eigenvalue: $\lambda - \frac{2}{3} \lambda - \frac{2}{3} = 0$

$6\lambda^2 - 3\lambda - 4 = 0$

$$
\lambda = \frac{3 \pm \sqrt{9 - 4(6)(-4)}}{12} = \frac{3 \pm \sqrt{9 - 4(-24)}}{12} = \frac{3 \pm \sqrt{9 + 96}}{12} = \frac{3 \pm \sqrt{105}}{12}
$$

Dominant eigenvalue is $1.10$

Corresponding eigenvector is

$$
\begin{pmatrix}
0 & 2 \\
\frac{2}{3} & \frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} = 1.10
\begin{pmatrix}
x \\
y
\end{pmatrix}
$$

$$
\begin{align*}
2y &= 1.1x \\
\frac{2}{3}x + \frac{1}{2}y &= 1.1y
\end{align*}
$$

$y = 0.55x$

$\begin{pmatrix}
100 \\
55
\end{pmatrix}$ is an eigenvector

1) Long-term Growth rate is $10\%$ and $65\%$ hatchlings...
4. Here is a system of differential equations

\[
\frac{dx}{dt} = -3x + 4y \quad \text{where} \quad \mathbf{u}_0 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}
\]

\[
\frac{dy}{dt} = -6x + 7y
\]

Write this as a matrix differential equation.

\[
\frac{d\mathbf{u}}{dt} = \begin{pmatrix} -3 & 4 \\ -6 & 7 \end{pmatrix} \mathbf{u} \quad \text{on} \quad \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -6 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]

\[+2\]

Find \(x(t)\) and \(y(t)\)

\[x(t) = \frac{8e^{3t} - 9e^{7t} + 1}{3t - 9e^{7t} + 1} \quad \text{and} \quad y = \frac{3t - 9e^{7t} + 1}{12e^{3t} - 9e^{7t} + 1}
\]

\[\begin{pmatrix} 3t - 9e^{7t} + 1 \\ 12e^{3t} - 9e^{7t} + 1 \end{pmatrix} \]

\[\begin{pmatrix} 8e^{3t} - 9e^{7t} + 1 \\ 3t - 9e^{7t} + 1 \end{pmatrix} = (\begin{pmatrix} -3 & 4 \\ -6 & 7 \end{pmatrix}) \begin{pmatrix} x \\ y \end{pmatrix}
\]

\[\begin{pmatrix} -3 & 4 \\ -6 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[-3x + 4y = x - 4x + 4y = 0 \quad -6x + 7y = y - 6x + 6y = 0\]

\[\begin{pmatrix} -3 & 4 \\ -6 & 7 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]

\[
\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \hat{v}
\]

\[
\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[-1 = 2\alpha + \beta \quad 3 = 3\alpha + \beta
\]

\[
\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \hat{v}
\]

\[
\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[-1 = 2\alpha + \beta \quad 3 = 3\alpha + \beta
\]

\[
\hat{u}(t) = 4e^{3t} \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 9e^{7t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[+2\]
5. A yellow perch population is a species of freshwater fish. Suppose a population of interest lives to a maximum age of 3 years. Only 5% of age 1 individuals survive to age 2 and 20% of age 2 individuals survive to age 3. Both age 2 and age 3 fish produce 65 and 110 age 1 offspring, respectively.

\[
\begin{pmatrix}
0 & 65 & 110 \\
.05 & 0 & 0 \\
0 & .2 & 0
\end{pmatrix}
\]

The matrix above represents the system of difference equations.

The dominant eigenvalue for this matrix is 1.9 and an associated eigenvector is\( \begin{pmatrix} .969 \\ .026 \\ .002 \end{pmatrix} \).

In the long run, how will the population of yellow perch change and what percentage of the population will be yellow perch age 1? Use a sentence (or sentences) to answer the question. Explain how you get your answer.

{In the long run growth rate is \( .9 \), (\( 1 + .9 = 1.8 \)) this pop is growing very rapidly and in the long run about 97% of the population will be in stage 1 (age 1).}