Instructions:

1. First read all the questions, then commence with the questions you find easiest.
2. There are 5 questions.
3. Each question carries a weight of 10 points to a total of 50 points.
4. No books or notes are allowed.
5. Calculators are allowed.
6. Good Luck!
1. Classify the linear systems

\[ \begin{align*}
\dot{x} &= 2x + y \\
\dot{y} &= x + ay
\end{align*} \]

for all values of \( a \).
2. Is the system

\[
\begin{align*}
\dot{x} &= y (x^2 - 1) \\
\dot{y} &= y^2 - 1
\end{align*}
\]

gradient or does it have a conserved quantity? Determine the nullclines, fixed points, and draw a phase portrait.
3. Show that the system

\[
\begin{align*}
\dot{x} &= x \left( 5 - x - \frac{y}{1+x} \right) \\
\dot{y} &= y \left( \frac{x}{1+x} - \frac{1}{16}y \right)
\end{align*}
\]

leaves the first quadrant invariant and has a unique unstable fixed point there. (After you have shown that the fixed point is unique you can use that it has first coordinate \( x = 1 \).) Show that there is limit cycle by using a trapping region that is a rectangular box.
4. Find the fixed points for the system

\[
\begin{align*}
\dot{x} &= \mu x + 2y - 2x^2 \\
\dot{y} &= x + \mu y
\end{align*}
\]

Decide their type and classify all possible bifurcations.
5. For which $\mu$ does the system
\[
\begin{align*}
\dot{x} &= \mu x + y - xy^2 \\
\dot{y} &= -x + \mu y + x^2
\end{align*}
\]
not have any periodic orbits. Determine and classify the bifurcations at the origin.
Blank