Homework 1

We recall that $C([0,1])$ denotes the space of continuous functions $f : [0,1] \to \mathbb{R}$.
Justify every answer.

**Exercise 1** For each admissible set $A$ and functional $J : A \to \mathbb{R}$ below, answer the following questions:

(i) What is a lower bound of $J$ over $A$?

(ii) What is the infimum of $J$ over $A$?

(iii) Does $J$ achieve its infimum over $A$? If yes, provide a minimizer. If no, explain why.

a. $A = C([0,1]), J(f) = \int_0^1 (f(x) - 1)^2 \, dx$

b. $A = \{ f \in C([0,1]) \mid f \geq 0 \text{ and } f(0) = 1 \}, J(f) = \int_0^1 f(x) \, dx$

**Exercise 2** Determine whether the given set constitutes a vector space under the usual operations (addition, multiplication by a scalar) for functions.

a. The set $A = \{ f \in C([0,1]) \mid f(0) = 1 \}$

b. The set $A = \{ f \in C([0,1]) \mid f(1) = 0 \}$