Math-245A: Real Analysis
Syllabus
Tim Austin

Topic of the course

The basic setting for real analysis is spaces endowed with certain kinds of extra structure that generalize familiar notions from calculus in $\mathbb{R}^n$: principally, distance, length, area and volume. Calculus itself studies functions on $\mathbb{R}^n$ and related objects, and analysis seeks to make the methods of calculus rigorous and as general as possible.

Undergraduate analysis begins the rigorous study of these notions, based on metrics as an abstraction of distance and using Riemannian integration to define area and volume.

During the 20th century, various deficiencies with those definitions became clear. In particular, they are not as general as one would wish:

- Some mathematically natural examples of sets cannot be assigned an area or volume using Riemann integration;
- Some operations on functions are difficult to understand in terms of Riemann integration, even if all the functions in question are Riemann integrable;
- Some natural and useful notions of ‘closeness’ between mathematical objects cannot be described by a metric.

More powerful and general notions were developed to overcome these problems. Math-245A is mostly about measure theory, which provides a much more satisfactory notion of area and volume, first in $\mathbb{R}^n$ and then in all sorts of other spaces. Using measure theory, we also define and study the Lebesgue integral, which generalizes the Riemann integral and fixes its most pressing problems.
(Math-245A will assume familiarity with the basic theory of metric spaces. In Math-245B we move beyond them to study topological spaces, and we introduce important metrics and topologies on spaces of functions as a way of describing the ‘size’ or ‘regularity’ of a function and the ‘convergence’ or ‘approximation’ of functions.)

**Prerequisites**

Prior experience with real analysis is essential. As a rough guide, I will assume the contents of the UCLA courses math-131AH and math-131BH. In addition to knowing those contents, you must be quite fluent in the nuts and bolts of reasoning in mathematical analysis, particularly $\varepsilon$-$\delta$ arguments. At the level of math-245A I will not always present these in detail.

I will go over pre-requisite knowledge in more detail in the first class. You can also look at Chapter 1 of Bass or Chapter 0 of Folland for an idea of the pre-requisites (note: from Folland I won’t need all of the material about set theory, well-orderings and the axiom of choice until 245B).

**My philosophy regarding homeworks, exams and assessment**

For a course such as 245A, I believe that homeworks are of huge value for instruction, but very little value for assessment. The 10% contribution of the homeworks to your overall grade is just a modest incentive to do them. I strongly encourage you to collaborate on homeworks, but the work you submit must be your own.

Homework questions for grading will mostly be taken from Folland’s or Bass’s book. They are chosen to make you work with and understand the results we are covering in class, and should mostly be of moderate difficulty.

I also believe that the exams should be a test of *competence* more than *clev-erness*. For this reason, the exam questions will be a lightly modified selection of the homework questions!

Homeworks will be graded quite strictly, because their main purpose is to give you practice. The grading of the same questions will be less strict on the exams. Most problems will ask for proofs or the construction of examples. The homework grading will take into account presentation as well as correctness: show all working in a logical order, and use complete sentences where prose is called for. Part of a graduate training in mathematics is the ability to present mathematical reasoning clearly and completely. Please don’t panic if your homework grades
seem low – it need not mean that you’re doing badly! If you want to discuss your progress, feel free to contact me.

Some homeworks may also contain starred problems. These are not for credit, have no deadline, will not be graded, and will not appear on the exam. They may range over wider material or be more difficult than usual. These questions are your way to go deeper into the material and challenge yourself. I am happy to discuss them at length in person.

**Feedback**

You can give anonymous feedback on any aspect of the course, at any time, using a module on the course CCLE site. Very specific feedback is often the most helpful, even if it cannot be followed to the letter. Feedback which says, “The lectures go too fast” is useless, because the topics to be covered and the number of lecture hours are both fixed. Much more helpful feedback might say “Go faster on topic X and slower on topic Y”.