Let \((X, \rho)\) be a metric space. A subset \(E \subset X\) is **precompact** if its closure \(\overline{E}\) is compact.

**Proposition 0.1.** If \((X, \rho)\) is complete and \(E \subset X\), then the following are equivalent:

\[
\begin{align*}
  a) & \quad E \text{ is precompact;} \\
  b) & \quad E \text{ is totally bounded;} \\
  c) & \quad \text{every sequence in } E \text{ has a subsequence which converges to some point in } X.
\end{align*}
\]

If you don’t recall these facts, try supplying your own proof, using the various equivalent characterizations of compactness itself.

Also, try seeing which of the above implications still hold if we drop the assumption that \((X, \rho)\) is complete. (Perhaps most interesting: show that (b) no longer implies (a).)