General description
In this course, you will learn how to design, analyze, and run mathematical models, particularly ones that apply to the study of dynamical systems in biology. We will start with single-variable and multivariable differential equation models. We will develop such models for dynamical processes in ecology, physiology, and other subjects in which quantities change with time, and we will learn how to analyze the behavior of these models. Along the way, we will learn the relevant parts of calculus, as well as how to run these models on a computer.

Instructor
Michael Andrews
Office MS 6322
Email mjandr@math.ucla.edu
If you email me, please mention LS 30A in your subject.

Lecture
MWF 2:00pm-2:50pm in La Kretz Room 110

Office hours
Wednesday 4:00pm-5:00pm, Thursday 10:00am-12:00pm.
They will be held in Life Sciences 2325.

Teaching assistants
Vennis Hong (vennishong@ucla.edu)
Junjie Shen (junjieshen@ucla.edu)
Aaron Zhang (zhjwzhang@g.ucla.edu)

Textbook
Garfinkel, Shevtsov, and Guo.

A free PDF of the book is available online at:

In order to access the book, you will need to be on UCLA campus or else you’ll need to use a UCLA VPN.
Grading  Your overall score will be determined by the better of the following two schemes:

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<th>Scheme 1</th>
<th>Scheme 2</th>
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<tr>
<td>Clicker</td>
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<tr>
<td>Homework</td>
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<td>Labs</td>
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<td>Lab Exam</td>
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<td>Worse Midterm</td>
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<td>Final</td>
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The more advantageous scheme will be applied automatically. See below (page 5) for Life Sciences’ policy on grade cutoffs.

Homework  Homework problems from the textbook will be assigned each week. They will be announced in class and posted on the course website. They will be collected in class, usually on Fridays. They will be graded and returned to you as promptly as possible. **Late homework is not accepted**, but your lowest two homework scores will be dropped.

Exams  Midterm 1: 6:00pm-7:50pm on Monday, January 28.
Midterm 2: 6:00pm-7:50pm on Wednesday, February 20.
Final: 3:00pm-6:00pm on Tuesday, March 19.

Please note that the midterms are scheduled in the evening, **outside of class time**. Please make sure that your schedule is clear on those days at that time! The locations of the exams will be announced later.

No notes or books may be used during the exam. **There will be no make-ups for missing a midterm.** If you have to miss a midterm your grade will be computed using the second scheme above. Per university regulations, you **must take the final in order to pass the course.** Make-ups for the final exam are permitted only under exceptional circumstances, as outlined in the UCLA student handbook. Please bring a photo ID to each exam.

You are strongly advised to bring a calculator to these exams, however it may not be a graphing calculator, nor a programmable calculator, nor any device capable of communicating with another device (such as a cell phone or tablet). A basic four-function calculator may suffice, but you might prefer a non-graphing scientific calculator. (You will also probably find this useful for exams in your chemistry and physics classes.)

Academic Integrity  You must not cheat, because cheating is morally wrong. For more details, see: [https://www.deanofstudents.ucla.edu/Student-Conduct](https://www.deanofstudents.ucla.edu/Student-Conduct)

You are permitted (and encouraged) to collaborate on your homework, but you must write it up yourself. You must name your collaborators on your homework.
Course website  All the materials for this course will be stored on our CCLE site: https://ccle.ucla.edu/course/view/19W-LIFESCI30A-2.

Official mode of communication  The official mode of communication for all practical matters pertaining to this course are email announcements sent from our CCLE site. It is your responsibility to monitor your email for these announcements, and (just to be safe) occasionally check the announcements on our CCLE site. All the announcements will be stored there.

Schedule  You can find a tentative schedule of topics over the page.

Clickers  In this course, we will put a great emphasis on interactive learning. In particular, throughout the lectures, I will be posing you questions about the material we are discussing, and you will record your answers by using either one of iClicker+ or iClicker2.

iClicker2 is the newer model, but either one will work. You must acquire a clicker before our first lecture on January 7. Make sure to register your clicker on our CCLE site (top-right corner, “Student Registration”) as soon as possible, so that your clicker responses will be correctly counted towards your grade. You will not be penalized for incorrect answers to clicker questions, so there should be no pressure here. But, you will be marked off for not answering at all, so you must attend the lectures and use your clicker to get credit for this aspect of the course.

Labs  Throughout this course, we will use mathematics software called SageMath (or Sage for short), via an online interface called CoCalc, to explore many of the mathematical concepts and models that we develop. Each week, you will attend a two-hour computer lab session, in which you will work individually or in small groups to solve a series of problems using Sage. This will require some computer programming, but all of the necessary concepts and skills will be taught in the lab. No prior programming experience is assumed. Any work that you do not complete in lab and any additional writing up that you need to do before turning it in can be done at home or during open lab hours. Labs will generally be due one week after they are started (i.e. typically right before the following week’s lab section begins).
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<tr>
<th>Date</th>
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<td>States, state space, trajectories.</td>
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<td>WEEK 2</td>
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Course grading

We are committed to making sure the assessment of your learning in this course is comprehensive, fair, and equitable. Your grade in this class will be based on the number of points you earn out of the total number of points possible and is **not based on your rank** relative to other students. Furthermore, grades are assigned without strict limits on the proportion of each letter grade given in the course.

**Cutoffs:**

If the class mean is 75% or higher, letter grades will be based on a straight scale using the following thresholds for grade cut-offs: **A** range from 90-100%, **B** range from 80-89.9%, **C** range from 70-79.9%, **D** range from 60-69.9%, and **F** given to 59.9% or lower. Within each letter grade range, a minus will be assigned to the bottom three percentage points and a plus will be assigned to the top three percentage points (e.g. 80-82.9% is a B−, 87-89.9% is a B+). If the class mean is lower than 75%, the scale will be adjusted to compensate (e.g. 89% may become an A−). **Under no circumstances will grades be adjusted down.** You can use the straight grading scale as an indicator of your minimum grade in the course at any time during the course. You should keep track of your own points so that at any time during the quarter you may calculate your minimum grade based on the total number of points possible at that particular time. If and when, for any reason, you have concerns about your grade in the course, please come and speak with us or your TA so that we can discuss study techniques or alternative strategies to help you.

**Regrading policy:**

Any request for a regrade must be made within one week of the assignment being returned to you. If you think there has been a simple addition error on your assignment then write a note explaining the error, attach this to the front of the assignment and turn it into the LS Core office. If the error is confirmed, then the points will be added to your score before the end of the quarter. If you think there was an error in grading that is not an addition error, write a brief note explaining why you think more points should be awarded, attach this to the front of the assignment and turn it into the LS Core office. In the event that you do turn in a regrade, you should make a photocopy for your own records. All regrades (except addition errors) will be done at the end of the quarter after we have calculated final grades. If the points you request will affect your final grade, then we will reevaluate the assignment for the contested points.
Opt-in for Gradescope

In this course, we would like to use an online platform called Gradescope (www.gradescope.com) to grade your exams. This platform streamlines the grading process in large-enrollment courses for exams with open-response questions, allowing for more consistent and fair grading. The points breakdown for the score you receive on an exam graded using Gradescope will be transparent because you will be able to see the comments explaining what you did correctly or incorrectly in connection with the points (sometimes awarded as partial credit) assigned for a given question.

All exams are scanned and uploaded to the Gradescope website by your TAs. Your instructors and TAs will access your responses and score them online. **Materials uploaded to Gradescope will contain your name, UCLA email address, and the last 6 digits of your student ID number as well as your responses to questions on the exam.** Feedback in the form of numerical scores and written comments will be stored on the site and returned to you electronically with notifications given via the email address associated with your Gradescope account.

Using this platform for feedback on your exams is optional. If you would prefer that your exams not be uploaded to Gradescope, then your answers instead will be scored by hand and returned to you to view as a hard copy (on paper).

To comply with FERPA guidelines, please complete the Permission to Release Education Record Form on CCLE indicating whether or not you will allow us to score your exams using Gradescope. For information about Gradescope’s privacy policy, please see https://gradescope.com/privacy.

If you opted into using Gradescope, **you will need to create an account on Gradescope and enroll in our course on Gradescope.** In order to do this, you will need the following code: 

???

Please complete the permission survey by January 21. If you opted into using Gradescope, please complete the Gradescope registration by January 23.
Academic Integrity - A Bruin’s Code of Conduct

UCLA is a community of scholars committed to the values of integrity. In this community, all members including faculty, staff, and students alike are responsible for maintaining the highest standards of academic honesty and quality of academic work. As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. When accusations of academic dishonesty occur, the Office of the Dean of Students investigates and adjudicates suspected violations of this student code. Unacceptable behaviors include cheating, fabrication, plagiarism, multiple submissions without instructor permission, using unauthorized study aids, or facilitating academic misconduct.

Please review our campus’ policy on academic integrity in the UCLA Student Conduct Code: https://www.deanofstudents.ucla.edu/Student-Conduct-Code.

Once referred to the Office of the Dean of Students, allegations of academic dishonesty can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

We cannot make exceptions to our campus’ policy on academic integrity, and as we hopefully have communicated effectively here, penalties for violations of this policy are harsh. Please do not believe it if you hear that “everyone does it”. The truth is, you usually don’t hear about imposed disciplinary actions because they are kept confidential. So our advice, just don’t do it! Let’s embrace what it means to be a true Bruin and together be committed to the values of integrity.

Examples of academic dishonesty possible in LS Core courses:
With respect to our course, examples of academic dishonesty include giving answers on assignments to someone else, receiving answers from someone else, turning in any written work that is not your own for points in our course, copying passages from websites, copying passages from your or any other textbook on any graded material in the course, or bringing a classmate’s clicker to class to get participation points for them when they are absent. If you engage in these types of unacceptable behaviors, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam (e.g., using notes, using cell phones or other smart devices to send, receive, or research an answer, looking on someone else’s exam, allowing someone else to look at your exam for answers), then you will receive a score of zero for the entire exam.

Exams:
No cell phones, smart watches, or similar types of devices are allowed during exams. Accordingly, you may not use cell phones as a clock to keep time or as a calculator. Please leave these items in your backpack and turned off or submitted to a TA/exam proctor. Be prepared to leave your backpacks and personal items (including hats) at the front of the room when taking your exams.

Enrollment:
In the event that we are at maximum enrollment capacity and you would like to enroll in this course, please monitor the Schedule of Classes in case someone drops the course. If you have other
enrollment concerns, please go to the LS Core Office in 2305 Life Sciences Building where you can talk to a Student Affairs Officers (SAOs) for the Life Science Core Education Department. We and the SAOs are unable to provide students permission to enroll (PTE) numbers.

**Changing Discussion/Lab Sections:**
Participation in discussion and lab sections is required for this course, and you must attend the section in which you are enrolled.

Please note that you are not permitted to switch enrollment in discussion/lab sections after the study list deadline (Friday of Week 2). If you would like to switch sections during the first week but there are no spots available in the desired section, you need to find another student who agrees to switch sections with you. To make the switch in discussion/lab sections official with the registrar, you both will need to go to the LS Core Office in 2305 Life Sciences Building and discuss your intention to switch sections in person.

**Absences in Discussion/Lab Sections (including required documentation policy):**
If there is an extenuating circumstance and you miss a discussion/lab section, please provide verifiable documentation to the LS Core office (within 7 days of your absence) to explain your absence, the basis of which is subject to their approval.

**Absences in Lecture or Exams (including required documentation policy):**
If you miss a lecture or exam, please provide verifiable documentation to the LS Core office (within 7 days of your absence) to explain your absence, the basis of which is subject to their approval.
Our Inclusive Learning Environment

UCLA values diversity and inclusion. We expect everyone in this class to contribute to a respectful, welcoming, and inclusive environment to support the learning of all other members of the class. If there are aspects of the instruction or design of this course that result in barriers to your inclusion or accurate assessment or achievement, please notify us.

Center for Accessible Education (CAE):
Students needing academic accommodations based on a disability should contact the Center for Accessible Education (CAE) at (310) 825-1501 or present in person at Murphy Hall A255. CAE will assess all requested accommodations and communicate appropriately with us (your instructors). Any students with CAE approval for proctoring arrangements during exams will need to please inform us (or your TA) prior to the date of the exam. When possible, students should contact CAE within the first two weeks of the quarter to allow reasonable time to coordinate accommodations. For more information, please visit the CAE website: https://www.cae.ucla.edu.

Counseling and Psychological Services (CAPS):
Resources are available to foster the well-being of all UCLA students as they pursue their academic goals. Any student who finds themselves in immediate distress, please call Counseling and Psychological Services (CAPS) to speak directly with a counselor 24/7 at (310) 825-0768, or please call 911. For more information, please visit the CAPS website: https://www.counseling.ucla.edu.

Student Resources for Support and Learning

UCLA has a multitude of resources available to all students. Many of these resources are listed below (alphabetized by name), and we encourage students to explore them as needed.

- **Academic Achievement Program** (AAP): This program advocates and facilitates the access, academic success, and graduation of students who have been historically underrepresented in higher education; informs and prepares students for graduate and professional schools; and develops the academic, scientific, political, economic, and community leadership necessary to transform society: https://www.aap.ucla.edu.

- **Academics in the UCLA Residential Community**: Free workshops on a wide variety of issues relating to academic & personal success; (310) 825-9315; https://reslife.ucla.edu (click on “academics”).

- **Bruin Resource Center**: Includes services for transfer students, undocumented students, veterans, and students with dependents; http://www.brc.ucla.edu.

- **Career Center**: Don’t wait until your senior year – visit the career center today! http://www.career.ucla.edu.

- **Center for Accessible Education** (Formerly Office for Students with Disabilities): Located in A255 Murphy Hall: (310) 825-1501, TDD (310) 206-6083; http://www.cae.ucla.edu.
• **Counseling and Psychological Services (CAP):** Located in Wooden Center West; students in distress may call to speak directly with a counselor 24/7 at (310) 825-0768, or may call 911; [http://www.counseling.ucla.edu](http://www.counseling.ucla.edu).
  
  – Commonly known as the “Red Folder”, this tool is intended to provide you with quick access to important resources for assisting students in distress (see, say, do): [https://ceils.ucla.edu/wp-content/uploads/sites/2/2016/08/911Guide.pdf](https://ceils.ucla.edu/wp-content/uploads/sites/2/2016/08/911Guide.pdf).

• **Dashew Center for International Students and Scholars:** Located in 106 Bradley Hall; (310) 825-1681; [http://www.internationalcenter.ucla.edu](http://www.internationalcenter.ucla.edu).

• **Dean of Students Office:** General resource for all Bruins. Learn about academic integrity issues and your first amendment rights. Get help if you’ve experienced rape or sexual assault. Report a bias incident, and much more. Located in 1206 Murphy Hall; (310) 825-3871; [http://www.deanofstudents.ucla.edu](http://www.deanofstudents.ucla.edu).

• **Lesbian, Gay, Bisexual and Transgender Resource Center:** Located in the Student Activities Center, B36; (310) 206-3628; [http://www.lgbt.ucla.edu](http://www.lgbt.ucla.edu).

• **Letters & Science Academic Counseling Service:** Located in A316 Murphy Hall; (310) 825-1965; [http://cac.ucla.edu](http://cac.ucla.edu).

• **Library:** Get help with your research, find study spaces, attend a workshop, rent a laptop, and more. Learn more at [http://www.library.ucla.edu](http://www.library.ucla.edu).

• **Student Legal Services:** Located in A239 Murphy Hall; (310) 825-9894; [http://www.studentlegal.ucla.edu](http://www.studentlegal.ucla.edu).

• **Undergraduate Writing Center:** Peer learning facilitators (PLFs) are undergraduates who understand the challenges of writing at UCLA. Scheduled appointment and walk-in options are available, see [http://wp.ucla.edu/wc](http://wp.ucla.edu/wc) for more information and to get assistance with your writing.

• **UCLA ONE:** This website ([https://uclaone.com/](https://uclaone.com/)) serves as UCLA’s interactive, online gateway for mentorship, professional networking, peer driven career advice, and exclusive job leads. (Similar to LinkedIn but for the UCLA community).
Integrity of Course Content

Please protect the integrity of all course materials and content. By enrolling in this course, you agree to honor this request. Be mindful of the hard work and time that our instructors and TAs in the LS Core put into creating course materials such as exam and quiz questions, worksheets, lecture videos, and Bruincasts. Please do not upload course materials not created by you onto third-party websites or share content with anyone who is not enrolled in our course. We are grateful for your cooperation in honoring this important request.

Learning Goals of LS30A

- Given a verbal description of interacting variables, understand how to make a differential equation model of a dynamical system, using the concepts of state space and tangent space.
- Given a differential equation, write Python-like code to simulate this model using Euler's method.
- Given verbal descriptions of dynamical systems, use the fundamental ideas of derivative and integral to describe their behavior.
- Given systems that exhibit bistability and switch-like behavior, be able to make models of these systems using the concept of positive feedback.
- Given a positive feedback system, be able to identify the parameters responsible for switch-like behavior.

Detailed Learning Outcomes of LS30A

1. **State and state space.** *Students should be able to:*
   - Explain what a state variable is and what its properties are;
   - Explain what state space is and draw or describe the state space for a given model;
   - Add vectors and multiply vectors by scalars;
   - Interpret scalar multiplication geometrically;
   - Explain what a trajectory is and convert between trajectories and time series, in both directions.

2. **Differential equations.** *Students should be able to:*
   - Explain how differential equations define the movement of state through state space;
   - Show how to build a model using differential equations by asking the questions “What makes $x$ go up?” and “What makes $x$ go down?” for each state variable.

3. **Vector fields.** *Students should be able to:*
   - Explain how differential equations define a vector field;
• Compute the change arrow for an arbitrary point in state space from a given model;
• Sketch sample trajectories from a vector field;
• Explain why trajectories are always tangent to the change arrows;
• Draw trajectories following the rules shown in class (such as trajectories are unique, they cannot cross, cannot attain an equilibrium point, tangent to change vectors, crossing the nullclines either horizontally or vertically (in 2-variable systems));
• Use Euler’s method (by hand and in Sage) to simulate a dynamical system;
• Explain the trade-offs involved in choosing a step size for Euler’s method.

4. The art of modeling. Students should be able to:

• Write models from assumptions using constant rates, constant per-capita rates, “X meets Y” terms, logistic terms, sigmoid and saturating functions.
• Distinguish between constant rates and constant per-capita rates;
• Build models for simple mechanical, chemical, ecological, epidemiological and other systems;
• Explain basic principles for modeling mechanical systems and build models for simple mechanical systems;
• Explain basic principles for modeling chemical reactions and build models for simple chemical reactions;
• Explain basic principles for modeling ecological models and build models for simple ecological models;
• Explain basic principles for modeling infectious diseases and build models for the spread of infectious diseases;
• Interpret a given model and explain the meaning of the parameters and the different terms of the differential equations.

5. Finding equilibrium points and nullclines. Students should be able to:

• Find equilibrium points for a given differential equation, or set of differential equations using the graphical method;
• Find and draw nullclines;
• Explain why equilibrium points always lie at the intersection of the nullclines.

6. Stability of equilibrium points. Students should be able to:

• Determine the stability of equilibrium points from computed change vectors in 1-D;
• Classify equilibrium points in 2D given a vector field;
• Use linear stability analysis to find the stability of equilibria in one dimension;
• Explain the relationship between linear stability analysis and vector fields.

7. Bifurcations. Students should be able to:

• Define “bifurcation” and use the over-under method to find where they occur;
• Draw and interpret bifurcation diagrams;
• Classify bifurcations in 1D as saddle-node, transcritical or pitchfork;
• Use a bifurcation diagram to create desired changes in a system’s behavior.

8. Oscillations and Feedback Loops. **Students should be able to:**

• Define “feedback”, “positive feedback” and “negative feedback”;
• Identify and classify feedback loops in verbal descriptions of systems and differential equations;
• Distinguish between fluctuations and oscillations and give examples of oscillations in physiological, chemical and other systems;
• Define “Hopf bifurcation” and describe what aspects of a system might be manipulated to bring one about.

9. Calculus Concepts. **Students should be able to:**

• Define “tangent”;
• Distinguish between average and instantaneous rates of change and explain how instantaneous rates of change might be approximated in a real-world situation;
• Compute derivatives of constant, linear and power functions;
• Explain the concept of a derivative in terms of rates of change, tangents to curves and linear approximations.
• Numerically approximate the area under a curve.

10. Programming. **Students should be able to:**

• Write a simple Python program using variables, lists, for loops, while loops, if statements and functions;
• Debug (find and fix) common programming errors;
• Implement Euler’s method in Sage;
• Use built-in tools in Sage to simulate differential equations and plot the results as both time series and trajectories;
• Formalize an intuitive description of a simple process, such as finding the parameter value at which a bifurcation occurs, and write a program to carry it out.