Math-245B: Real Analysis
Syllabus

Tim Austin

**Topic of the course**

Math 245B is the second part of the graduate-level real analysis sequence at UCLA. In my approach, measure theory is covered in the first part (245A), and then 245B is given to the basics of functional analysis.

Real analysis is largely concerned with functions and measures. The great insight behind functional analysis is that one can understand these kinds of objects better by considering entire *spaces* that contain them, rather than treating them ‘one at a time’. These ‘spaces of objects’ usually turn out to be vector spaces, but very ‘large’ ones: in particular, they are infinite dimensional. As a result, the structure and properties of these spaces depend not just on infinite-dimensional linear algebra, but also on notions of closeness or approximation for elements of these spaces. Formalizing these notions leads to the basic settings of *Banach spaces*, *topological vector spaces*, and *Hilbert spaces*.

Math 245B begins with an introduction to point-set topology, which largely generalizes students’ prior knowledge of metric spaces. Then it introduces these different classes of spaces, illustrating them with the basic examples that arise from continuous functions, measurable functions, or measures themselves.

**Prerequisites**

Prior experience with real analysis and measure theory is essential. I teach measure theory in 245A and then basic functional analysis in 245B. Most students will have taken 245A with me previously, and will ideally have earned at least a B in that course. Please contact me in person if you have not taken 245A but wish to take 245B.

In addition to 245A, some prior exposure to general set theory and point-set topology will be helpful in 245B. Students without that background should be
prepared to do some extra reading. I will go over some pre-requisite knowledge from set theory in the first class.

My philosophy regarding homeworks, exams and assessment

For a course such as 245B, I believe that homeworks are of huge value for instruction, but very little value for assessment. The 10% contribution of the homeworks to your overall grade is just a modest incentive to do them. I strongly encourage you to collaborate on homeworks, but the work you submit should be your own.

Homework questions for grading will mostly be taken from Folland’s or Bass’s book. They are chosen to make you work with and understand the results that we cover in class, and should mostly be of moderate difficulty.

I also believe that the exams should be a test of competence more than cleverness. For this reason, the exam questions will be a lightly modified selection of the homework questions!

Homeworks will be graded quite strictly, because their main purpose is to give you practice. The grading of the same questions will be less strict on the exams. Note: Since careful grading is very time-consuming, it may be impossible for the TA to grade all of the homework problems. In this case they will grade a subset that I choose. The subset of problems to be graded will not be revealed in advance, so please do all the problems.

Most problems will ask for proofs or the construction of examples. The homework grading will take into account presentation as well as correctness: show all working in a logical order, and use complete sentences where prose is called for. Part of a graduate training in mathematics is the ability to present mathematical reasoning clearly and completely. Please don’t panic if your homework grades seem low – it need not mean that you’re doing badly! If you want to discuss your progress, feel free to contact me.

Occasionally I assign some extra ‘starred’ problems, separately from the regular homeworks. These are not for credit, have no deadline, will not be graded, and will not appear on the exam. They may range over wider material or be more difficult than usual. These questions are your way to go deeper into the material and challenge yourself. I am happy to discuss them at length in person.

Feedback

You can give anonymous feedback on any aspect of the course, at any time, using a module on the course CCLE site. Very specific feedback is often the most helpful,
even if it cannot be followed to the letter. Feedback which says, “The lectures go
too fast” is useless, because the topics to be covered and the number of lecture
hours are both fixed. Much more helpful feedback might say “Go faster on topic
X and slower on topic Y”. 