Markov Logic Networks: Semantics and Learning

CS267A - Fall 2018
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Undirected Relational Graphical Models
Variable Elimination
Factors Multiplication

\[
\begin{array}{c|c|c}
A & B & f_1(A, B) \\
\hline
T & T & .3 \\
T & F & .7 \\
F & T & .9 \\
F & F & .1 \\
\end{array}
\times
\begin{array}{c|c|c}
B & C & f_2(B, C) \\
\hline
T & T & .2 \\
T & F & .8 \\
F & T & .6 \\
F & F & .4 \\
\end{array}
= \\
\begin{array}{c|c|c|c}
A & B & C & f_3(A, B, C) \\
\hline
T & T & T & .3 \times .2 = .06 \\
T & T & F & .3 \times .8 = .24 \\
T & F & T & .7 \times .6 = .42 \\
T & F & F & .7 \times .4 = .28 \\
F & T & T & .9 \times .2 = .18 \\
F & T & F & .9 \times .8 = .72 \\
F & F & T & .1 \times .6 = .06 \\
F & F & F & .1 \times .4 = .04 \\
\end{array}
\]
Summing out Variable from Factor

\[
\sum_a f_3(A, B, C) = f_3(a, B, C) + f_3(\neg a, B, C)
\]

\[
= \begin{pmatrix} .06 & .24 \\ .42 & .28 \end{pmatrix} + \begin{pmatrix} .18 & .72 \\ .06 & .04 \end{pmatrix} = \begin{pmatrix} .24 & .96 \\ .48 & .32 \end{pmatrix}.
\]
Factor Graphs

\[
P(X = x) = \frac{1}{Z} \prod_k \phi_k(x_{\{k\}}) \\
\sum_{x \in X} \prod_k \phi_k(x_{\{k\}})
\]
Log-Linear Models

\[ P(X = x) = \frac{1}{Z} \exp \left( \sum_j w_j f_j(x) \right) \]

Factor graphs where factors are made up of features $f$ and weights $w$. 
Statistical Relational Representations

Augment graphical model with relations between entities (rows).

**Intuition**

- Friends have similar smoking habits
- Asthma can be hereditary

**Markov Logic**

1.9 \[ \text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y) \]
1.5 \[ \text{Asthma}(x) \land \text{Family}(x,y) \Rightarrow \text{Asthma}(y) \]
Equivalent Graphical Model

- Statistical relational model (e.g., MLN)

\[ 1.9 \quad \text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y) \]

- Ground atom/tuple = \textbf{random variable} in \{true,false\} e.g., Smokes(Alice), Friends(Alice,Bob), etc.

- Ground formula = \textbf{factor} in propositional factor graph

\[
\begin{align*}
\text{Friends}(Alice,Alice) & \quad f_3 \\
\text{Smokes}(Alice) & \\
\text{Smokes}(Bob) & \\
\text{Friends}(Bob,Alice) & \quad f_2 \\
\text{Friends}(Alice,Bob) & \quad f_1 \\
\text{Friends}(Bob,Bob) & \quad f_4
\end{align*}
\]
How many ways are there to instantiate (ground) the free variables, such that formula i is satisfied in world x.
Inference Problem

Given:

MLN:
0.7 Actor(a) ⇒ ¬Director(a)
1.2 Director(a) ⇒ ¬WorkedFor(a,b)
1.4 InMovie(m,a) ∧ WorkedFor(a,b) ⇒ InMovie(m,b)

Database tables (if missing, then w = 0)

| Actor:   |  |  |
|----------|  |  |
| Name     | w |
| Brando   | 2.9 |
| Cruise   | 3.8 |
| Coppola  | -1.1 |

| WorkedFor: |  |  |
|------------|  |  |
| Actor      | Director | w |
| Brando     | Coppola  | 2.5 |
| Coppola    | Brando   | -0.2 |
| Cruise     | Coppola  | 1.7 |

Compute:

\[ P(\text{InMovie(GodFather, Brando)} = ??) \]
MLN Weight Learning

See note with derivations
Parameter Learning

- **Given:** A set of first-order logic formulas
  \[ w \text{ FacultyPage}(x) \land \text{Linked}(x,y) \Rightarrow \text{CoursePage}(y) \]

- **Learn:** The associated maximum-likelihood weights

\[ \frac{\partial}{\partial w_j} \log \Pr_w(db) = n_j(db) - \mathbb{E}_w[n_j] \]

- Count in databases Efficient
- Expected counts Requires inference

\[ \mathbb{E}_w[n_F] = \Pr(F\theta_1) + \cdots + \Pr(F\theta_m) \]
Structure Learning

- **Given:** A set of training databases
- **Learn:** A set of first-order logic formulas
  The associated maximum-likelihood weights
- **Idea:** Search, pathfinding.