Clicker question 1

Consider

\[ f: \{\text{Michael, Omie, Laura, Jaycie}\} \rightarrow \{\text{Meshuggah, Travis Scott, Bruce Springsteen, Ariana Grande, Sheck Wes}\} \]

with

\[
\begin{align*}
 f(\text{Michael}) &= \text{Meshuggah} \\
 f(\text{Omie}) &= \text{Travis Scott} \\
 f(\text{Laura}) &= \text{Bruce Springsteen} \\
 f(\text{Laura}) &= \text{Ariana Grande} \\
 f(\text{Jaycie}) &= \text{Sheck Wes}
\end{align*}
\]

Is this a function?

No. Laura is just one input: one first name. BUT there are two outputs corresponding to this name: Bruce Springsteen and Ariana Grande.

Here’s a picture explanation:

\[\begin{array}{c}
\text{Michael} \rightarrow \text{Meshuggah} \\
\text{Omie} \rightarrow \text{Travis Scott} \\
\text{Laura} \rightarrow \text{Bruce Springsteen} \\
& \rightarrow \text{Ariana Grande} \\
\text{Jaycie} \rightarrow \text{Sheck Wes}
\end{array}\]

You can see two arrows emanating from the name Laura. That is not allowed.

Alternatively, if we allowed \( f(\text{Laura}) = \text{Bruce Springsteen} \) and \( f(\text{Laura}) = \text{Ariana Grande} \), we’d have \( \text{Bruce Springsteen} = \text{Ariana Grande}!! \) This is definitely wrong. They seem a little different to me!
Clicker question 2

Consider

\[ f : \{\text{Michael, Omie, Laura, Jaycie}\} \rightarrow \{\text{Meshuggah, Travis Scott, Sheck Wes}\} \]

with

\[
\begin{align*}
f(\text{Michael}) &= \text{Meshuggah} \\
f(\text{Omie}) &= \text{Travis Scott} \\
f(\text{Jaycie}) &= \text{Sheck Wes}
\end{align*}
\]

Is this a function?

This tricked 86% of the class into saying “yes”. The rule defining the function looks reasonable. **BUT** the name Laura is in the domain, yet there is not a corresponding output. That is, there are 0 outputs, when there should be exactly 1. It is not a function.

Here’s a picture explanation:

\[
\begin{align*}
\text{Michael} & \rightarrow \text{Meshuggah} \\
\text{Omie} & \rightarrow \text{Travis Scott} \\
\text{Laura} & \\
\text{Jaycie} & \rightarrow \text{Sheck Wes}
\end{align*}
\]

There are no arrows emanating from the name Laura. That is not allowed.
Clicker question 3

Consider

\[ f : \{\text{Michael, Omie, Laura, Izzie, Jaycie}\} \rightarrow \{\text{Meshuggah, Travis Scott, Bruce Springsteen, Ariana Grande, Sheck Wes}\} \]

with

\[
\begin{align*}
  f(\text{Michael}) &= \text{Meshuggah} \\
  f(\text{Omie}) &= \text{Travis Scott} \\
  f(\text{Laura}) &= \text{Ariana Grande} \\
  f(\text{Izzie}) &= \text{Sheck Wes} \\
  f(\text{Jaycie}) &= \text{Sheck Wes}
\end{align*}
\]

Is this a function?
Yes. This is a function.
A relatively high percentage of people said “No” and they had good reasons for saying so…
Let’s see the picture:

\[
\begin{align*}
  \text{Michael} &\rightarrow \text{Meshuggah} \\
  \text{Omie} &\rightarrow \text{Travis Scott} \\
  \text{Laura} &\rightarrow \text{Bruce Springsteen} \\
  \text{Izzie} &\rightarrow \text{Ariana Grande} \\
  \text{Jaycie} &\rightarrow \text{Sheck Wes}
\end{align*}
\]

1. The first complaint was that there is no arrow to Bruce Springsteen: there is no input corresponding to Bruce Springsteen.
   While this is a valid observation, it does not stop \( f \) being a function.
   (Extra: it means \( f \) is not onto/surjective; you don’t need to know this.)

2. The second complaint was that there are two arrows to Sheck Wes: there are two inputs corresponding to Sheck Wes.
   While this is a valid observation, it does not stop \( f \) being a function.
   (Extra: it means \( f \) is not one-to-one/injective; you don’t need to know this.)
Final definition of the day

**Definition.** The *image* of a function \( f : X \rightarrow Y \) is the set of all elements \( y \) in \( Y \) which have an \( x \) mapping to them.

**Remark.** It can happen that the image of \( f : X \rightarrow Y \) is *not* all of the codomain \( Y \).

**Example.** The third clicker question: consider

\[
f : \{\text{Michael, Omie, Laura, Izzie, Jaycie}\} \\
\rightarrow \{\text{Meshuggah, Travis Scott, Bruce Springsteen, Ariana Grande, Sheck Wes}\}
\]

with

\[
\begin{align*}
  f(\text{Michael}) &= \text{Meshuggah} \\
  f(\text{Omie}) &= \text{Travis Scott} \\
  f(\text{Laura}) &= \text{Ariana Grande} \\
  f(\text{Izzie}) &= \text{Sheck Wes} \\
  f(\text{Jaycie}) &= \text{Sheck Wes}
\end{align*}
\]

The image of \( f \) is \( \{\text{Meshuggah, Travis Scott, Ariana Grande, Sheck Wes}\} \).

The codomain is \( \{\text{Meshuggah, Travis Scott, Bruce Springsteen, Ariana Grande, Sheck Wes}\} \).

Bruce Springsteen is in the codomain, but not the image.