A penta NFA is an NFA that accepts a word $w$ if there exist computation paths for $w$ such that one-fifth or more of the ending states are accepting. Show that penta NFAs and DFAs are equivalent.

Let $L$ be any language, and let $L_{alt}$ be the set of string in $L$ with every other character removed, i.e.

$$L_{alt} = \{ x \mid \exists y \in L \text{ such that } x_1x_2x_3\ldots = y_1y_3y_5\ldots \}$$

Show that if $L$ is a regular language, then $L_{alt}$ is regular.

Show that if $L$ is a regular language, then

$$L_n = \{ 0^k \mid k \text{ is a multiple of } n \}$$

is regular.

Show that if $L$ is a regular language, then

$$NOPREFIX(L) = \{ w \in L \mid \text{no proper prefix of } w \text{ is a member of } L \}$$

is regular.

Show that if $L_1, L_2$ are regular then

$$L_1 \text{ avoids } L_2 = \{ w \mid w \in L_1 \text{ and } w \text{ doesn't contain any string in } L_2 \text{ as a substring} \}$$

is regular.