1. Compute each of the following without a calculator:

   \[(a)\] \[\frac{(3^7)^4}{(3^4)^7}\] (Hint: Simplify the top and bottom.) \[\frac{3^{28}}{3^{28}} = 1\]

   \[(b)\] \[8^{5/3}\] (Hint: Use the fact that \[\frac{5}{3} = 5 \cdot \frac{1}{3}\] to rewrite this in two different ways. One of them is easy to compute.) \[8^{5/3} = \left(2^{3}\right)^{5/3} = 2^5 = 32\]

2. The conclusion from the end of Monday’s lecture was

   if \(f(x) = e^x\), then \(f'(x) = e^x\).

3. For this problem, let \(f(x) = x + e^{-x}\).

   \[(a)\] Find \(f'(x)\).

   \[f'(x) = 1 + \frac{d}{dx} e^{-x} \cdot (-1)\]

   \[= \frac{d}{dx} (1 - e^{-x})\]

   \[(b)\] \(f\) has one critical point. Find it.

   Set \(f'(x) = 0\):
   \[1 - e^{-x} = 0\]
   \[e^{-x} = 1\]
   \[x = 0\]

   \[(c)\] Review from 31A: Explain briefly the two ways to classify a critical point as a local maximum, local minimum, or neither.

   \[\text{One way: 1st derivatives only. Determine where } f'(x) > 0 \text{ (i.e. increasing) and } f'(x) < 0 \text{ (decreasing). If the function changes from inc. to dec. } \Rightarrow \text{ local max; dec to inc } \Rightarrow \text{ local min.}\]

   \[\text{No change } \Rightarrow \text{ neither.}\]

   \[\text{Another way 2nd derivatives. If } f''(x) > 0 \Rightarrow \text{ concave up } \Rightarrow \text{ local min.}\]

   \[f''(x) < 0 \Rightarrow \text{ concave down } \Rightarrow \text{ local max.}\]

   \[f''(x) = 0 \Rightarrow \text{ neither, inflection point}\]

   \[(d)\] Use either of the two methods to classify the critical point of \(f\).

   \[\text{1st derivatives only: } f'(x) = 1 - e^{-x}\]

   \[\text{Test points: }\]
   \[x = -1: f'(-1) = 1 - e^{-(-1)} = 1 - e < 0\]
   \[x = 0: f'(0) = 1 - e^{-1} = 1 - \frac{1}{2} > 0\]

   \[\text{dec to inc means local min at } x = 0\]

   \[\text{2nd derivative: } f''(x) = 1 - e^{-x}\]

   \[f''(0) = 1 - e^0 = 0 > 0\]

   \[\text{Concave up } \uparrow\]

   \[\text{local min at } x = 0\]
4. Rewrite as the logarithm of a single expression: $2 \ln(6) - 3 \ln(2)\]

$$2 \ln(6) - 3 \ln(2) = \ln(6^2) - \ln(2^3) = \ln\left(\frac{6^2}{2^3}\right) = \ln\left(\frac{9}{2}\right)$$

5. Consider the equation $\ln(x^2 + 1) - 3 \ln(x) = \ln(2)$

(a) Simplify the left-hand side into a logarithm of a single expression.

$$\ln(x^2 + 1) - \ln(x^3) = \ln\left(\frac{x^2 + 1}{x^3}\right)$$

(b) Then eliminate the logarithms by exponentiating both sides. You should be able to easily simplify the result into a polynomial.

$$e^{\ln\left(\frac{x^2 + 1}{x^3}\right)} = e^{\ln(2)}$$

$$\frac{x^2 + 1}{x^3} = 2$$

$$x^2 + 1 = 2x^3$$

$$2x^3 - x^2 - 1 = 0$$

(c) The polynomial you got in the above step is a cubic (degree 3), so it’s a bit tricky to solve. One of the roots is easy to guess. Once you know that one, how does that allow you to find the other roots?

Guess a value for $x$ so that $2x^3 - x^2 - 1 = 0$.

If we guess $x = 1$: $2(1)^3 - 1^2 - 1 = 2 - 1 - 1 = 0$. This means $x = 1$ is a factor of $2x^3 - x^2 - 1$.

What are the other factors? Use long division or synthetic division.

**Long division:**

$$\begin{array}{c|ccc}
\text{X-1} & 2x^2 + x + 1 \\
\hline
& 2x^3 - x^2 - x - 1 \\
& \underline{- (2x^3 - 2x^2)} \\
& \hline
& x^2 - 3x - 1 \\
& \underline{- (x^2 - x)} \\
& \hline
& -x - 1 \\
& \underline{- \quad (-x-1)} \\
& \hline
& 0 \\
\end{array}$$

**Synthetic division:**

$$\begin{array}{c|ccc}
2 & 2 & -1 & 0 & -1 \\
\hline
& 2 & 1 & 1 & 0 \\
\end{array}$$

Therefore $2x^3 - x^2 - 1 = 0 \Rightarrow (x-1)(2x^2 + x + 1) = 0 \Rightarrow x = 0$ or $2x^2 + x + 1 = 0$.

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 - 4(2)(1)}}{2(2)}$$

$$x = -1 \pm \sqrt{\frac{3}{4}}$$

Therefore, $x = -1 \pm \frac{\sqrt{3}}{2}$.