# Practice Midterm 1

Last Name: ________________________________

First Name: ________________________________

Last six digits of UID: _______________________

By signing below, you affirm that you have neither given nor received unauthorized help on this exam.

Signature: ________________________________

**Instructions:** Do not open this exam until instructed to do so. You will have 90 minutes to complete the exam. Please print your name and the last six digits of your student ID number above. You may not use books, notes, or any other material to help you. You may use a calculator, but not a programmable or graphing calculator. Please make sure your phone is silenced and stowed with your other belongings at the front of the room. You may use any available space on the exam for scratch work, including the backs of the pages. If you need more scratch paper, please ask one of the proctors.

Please do not write below this line.

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1. (10 points) Glycolysis, one of the ways that cells metabolize glucose, is a complex biochemical process consisting of ten reactions, which happen in sequence. In the third of these steps, fructose-6-phosphate is converted to another compound with the help of ADP, which acts as a catalyst. Since this is a complicated reaction, rather than just using a chemical reaction formula, use the following assumptions to model this reaction.

- Let $F$ be the concentration of fructose-6-phosphate and $A$ be the concentration of ADP.
- Fructose-6-phosphate is produced (from an earlier step of glycolysis) at a constant rate of 2.4 per minute.
- ADP is consumed by cellular processes at a per-mass rate of 0.85 per minute.
- When a molecule of fructose-6-phosphate comes into contact with two molecules of ADP, the reaction occurs, consuming the molecule of fructose-6-phosphate, and producing one new molecule of ADP, but not consuming the original two molecules of ADP. (They serve only as a catalyst.) Use a proportionality constant of 1.6 for this.

As usual, it is recommended you start with a diagram.
Question 1 continued... Last six digits of UID: ____________
2. (12 points) Clownfish are protandrous hermaphrodites, meaning that they may change from male to female at some point during their life cycle. Their colonies have a strict social hierarchy, consisting of non-reproductive juveniles \((J)\), who may eventually grow into breeding adult males \((M)\), who might later undergo sex reversal to become adult females \((F)\). Set up a model of the population of clownfish based on the following assumptions:

- For more juveniles to be born, an adult male has to come into contact with an adult female. Use a proportionality constant of 5.4 for this.
- Juveniles have a per-capita death rate of 43% per year, while adult males and females have a per-capita death rate of 26% per year.
- Juveniles only grow to become adult males when there is a scarcity of adult males. So assume the per-capita rate at which juveniles become adult males is proportional to the inverse of the adult male population, with a proportionality constant of 0.11.
- Similarly, an adult male will become female in the absence of other females to reproduce with. Assume that the per-capita rate at which males convert to female is proportional to inverse of the female population, with a proportionality constant of 0.83.

As usual, it is recommended you start with a diagram.

Question 2 continues on the next page...
Question 2 continued... Last six digits of UID: ____________

I have both male and female sexual anatomy!
3. The following system of differential equations models a small food web with three species: owls, which prey on rabbits, which in turn consume grass.

\[
\begin{align*}
X' &= 0.01XY - 0.001X^2 - 0.03XZ \\
Y' &= 0.3Y \left( 1 - \frac{Y}{500} \right) - 0.06XY \\
Z' &= 0.002XZ - 0.1Z
\end{align*}
\]

(a) (5 points) Which variable represents which population?

(b) (5 points) Explain briefly what each of the following terms in the equations likely models/represents:

i. \(-0.06XY\) in the \(Y'\) equation

ii. \(500\) in the \(Y'\) equation

iii. \(-0.1Z\) in the \(Z'\) equation

iv. \(0.002XZ\) in the \(Z'\) equation

v. \(-0.001X^2\) in the \(X'\) equation
4. (10 points) The following differential equations describe the position \((X)\) and velocity \((V)\) of a mass on a spring, with a nonlinear friction term.

\[
\begin{align*}
X' &= V \\
V' &= -X - 0.2V^2
\end{align*}
\]

Sketch the vector field of this system on the axes below. (Five or six vectors should be sufficient. Try to spread them throughout the state space.)
5. For parts (a), (b), and (c), say whether or not the relation described is a function. If not, explain why not.

(a) (2 points) \( DAYS = \{ \text{Mon, Tue, Wed, Thu, Fri, Sat, Sun} \} \)
\( PEOPLE = \{ \text{people living in Westwood} \} \)
\( f: DAYS \rightarrow PEOPLE \) defined by
\[ f(d) = \text{the people in Westwood who were} \]
\[ \text{born on day of the week } d \]

(b) (2 points) \( STUDENTS = \{ \text{students at Hogwarts} \} \)
\( HOUSES = \{ \text{Gryffindor, Hufflepuff, Ravenclaw, Slytherin} \} \)
\( sortinghat: STUDENTS \rightarrow HOUSES \) defined by
\[ sortinghat(w) = \text{the house that witch/wizard } w \text{ is assigned to} \]

(c) (2 points) \( L = \{ \text{straight lines in } \mathbb{R}^2 \} \)
\( Y = \mathbb{R} \)
\( int: L \rightarrow Y \) defined by \( int(l) = \text{the } y\text{-intercept of line } l \)

(d) (4 points) Each employee of a company earns a certain salary. This defines a function
\( salary: \{ \text{employees} \} \rightarrow \mathbb{R}_+ \), defined by \( salary(p) = p\text{'s salary} \)

Income tax is automatically deducted from each person’s salary, so that the amount of money they get to keep is 80% of their actual salary. Describe this second thing as a function. (Be sure to specify its domain and codomain.) What does the composition of these two functions represent?
6. (10 points) Identify at least one feedback loop in this system. Is it a positive or negative feedback loop? Justify your answer by drawing a feedback diagram.

\[
\begin{align*}
X' &= 3Y - Z \\
Y' &= X - 4Z \\
Z' &= X + 2Y \\
W' &= 2X + Y
\end{align*}
\]
7. (8 points) The vector field below was created from a model of the populations of moose ($M$) and wolves ($W$) in a region of Isle Royale, Michigan. Suppose that initially, there are 70 moose and 5 wolves in the region. Sketch an approximate trajectory starting from this initial state (you may sketch it directly on the vector field), and then give a rough sketch of what the corresponding time-series might look like.