Homework #4

1.1.1) In-Text

See back page for graph

A-B) The sharks eat the tuna and the shark population grows, behind the tuna population.

B-C) Eventually, the Tuna population diminishes until the state is such that there are high shark, and few tuna.

C-D) At this point, the Shark population declines because low tuna leads to few sharks being born.

D-E) The pressure is now off of the tuna, and they begin to re-populate. Consequently, the shark population growth follows, albeit delayed, and the cycle continues.

1.1.2) In-Text

Positive feedback, also known as “reinforcing feedback”, means that a positive value of a variable will lead to an increase, and a negative value will lead to a decrease. An example of this is population growth (given that there are enough resources, and space). The more animals that are born, the more there are to give birth to even more animals. Another example of this is when a student studies for an exam, does poorly, and stops trying. Consequently, they continue to do poorly, potentially even worse.

Negative feedback can be determined when a positive value of a variable leads to a decrease, or when a negative value leads to an increase. A classic example, involves insulin and glucose in the bloodstream. Intake of glucose causes the pancreas to secrete more insulin, which then lowers the level of glucose, by helping the glucose to be metabolized in the body.

1.2.1 In-Text

A function describes the relationship between input and output, where each input has exactly one output (never none, and never more than one).

For instance:

1) The amount taxed is a function of income. Say \( f(x) = 0.1 \times x \), where \( x \) is income, and \( y = f(x) \) is the amount taxed. This abides by the above definition of a function.

2) The item that comes out of a vending machine, is a function of the exact button combination pressed on the machine.

1.2.2 In-Text

Simply give one of the “Hot Beverage” choices two prices. For example Mocha can map to a price of $3.45 AND $0.99, making it no longer a function.

1.2.3 In-Text

From Figure 1.15, we see that \( g(3) = 3 - 1 = 2 \) and \( g(4) = 4 - 1 = 3 \). Thus, \( g(x) = x - 1 \).

1.2.4 In-Text

\( f(x) = x + 2 \), domain \( x \): \( \{3, 4\} \).

\( g(x) = x - 1 \), domain \( x \): \( \{3, 4\} \).
1.2.5 In-Text

\[ g(x) = \frac{2}{x-5} \]

Possible domains:
1) All real numbers greater than 5.
2) All integers except 5.
3) \([8, 10]\)

Generally, you want to pick the domain to make physical sense, in the context of what relationship the function is describing.

1.2.6 In-Text

The general form is written as: function name: \{domain\} -> \{codomain\} pronounced “f takes X to Y”

Menu: \{Drinks\} -> \{Prices\}

Vending Machine: \{Buttons\} -> \{Item\}

1.2.7 In-Text

We are told that RNA codon AUC translates to both serine (60% of the time) and histidine (40% of the time). Thus, martian gene expression is not a function because AUC maps to two potential amino acid codons (not just one).

1.2.8 In-Text

a) Let x be the menu item and let y be the set price.

\[ f: \{\text{menu item}\} \rightarrow \{\text{price}\}, \text{ in other words } x \rightarrow y \]

\[ g: \{\text{price}\} \rightarrow \{\text{total cost}\}, \text{ in other words } y \rightarrow g(y) \]

So \( g(y) = 1.1 f(x) \) is the total cost.

Mocha total cost : \( g(3.45) = 1.1 \times 3.45 = 3.795 \)

Latte total cost : \( g(3.15) = 1.1 \times 3.15 = 3.465 \)

b) Essentially, \( g \) composition \( f \) means that we copy down the \( g \) function, replacing \( y \) with the \( f(x) \) function.

\[ g \circ f(x) = g(f(x)) = 1.1 f(x) = 1.1 \ y \]

where \( y \) is the set price and \( x \) was the menu item.