Homework 1 Solutions
(40 points)

**Exercise 1.1.1**  Copy two full cycles of the predator–prey oscillation time series in Figure 1.2 and label the point at which each of the processes described in the above paragraph is occurring.

**Exercise 1.1.2**  Come up with another example of a positive feedback loop and another example of a negative feedback loop.

Positive feedback loop:

When you’re productive, you tend to be happier with yourself. When you’re happy, you are more motivated to get work done.
Negative feedback loop:

Studying more leads to better grades on assignments, which can make students cocky about the class. They think they are doing well enough, so they don’t study as hard, which then leads to poorer grades. They then start panicking about their performance in the class and start studying harder.

**Exercise 1.2.1** Come up with two more everyday examples of functions. Briefly explain what makes each example a function.

Example 1: Names are a function of ID numbers. Each ID number is unique and matches to only one name. However, ID numbers are not a function of names because multiple people can have the same name.

<table>
<thead>
<tr>
<th>ID Number</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>21366</td>
<td>John Smith</td>
</tr>
<tr>
<td>43341</td>
<td>Laura Brown</td>
</tr>
<tr>
<td>89421</td>
<td>John Smith</td>
</tr>
</tbody>
</table>

Example 2: Population sizes are a function of cities. Each city can have only 1 population size. Cities, however, are not a function of population sizes because multiple cities can have the same population. (note: the following numbers are made up for demonstration purposes)

<table>
<thead>
<tr>
<th>City</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>LA</td>
<td>100,000</td>
</tr>
<tr>
<td>NY</td>
<td>100,000</td>
</tr>
<tr>
<td>TX</td>
<td>85,000</td>
</tr>
</tbody>
</table>
**Exercise 1.2.2** Modify the menu in Figure 1.11 so it no longer depicts a function.

<table>
<thead>
<tr>
<th>HOT BEVERAGES</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mocha</td>
<td>3.45</td>
</tr>
<tr>
<td>Cappuccino</td>
<td>3.45</td>
</tr>
<tr>
<td>Macchiato</td>
<td>2.45</td>
</tr>
<tr>
<td>Latte</td>
<td>3.15</td>
</tr>
<tr>
<td>Americano</td>
<td>3.75</td>
</tr>
<tr>
<td>Espresso</td>
<td>2.95</td>
</tr>
</tbody>
</table>

**Exercise 1.2.3** Use the two input–output pairs on the right side of Figure 1.15 to write the function $g$.

\[ g(3) = 2 \]
\[ g(4) = 3 \]
Exercise 1.2.4  Write the functions $f$ and $g$ in Figure 1.15 in function notation using formulas.

\[
\begin{align*}
\text{input } 3 & \quad \text{input } 4 & \quad \text{input } 3 & \quad \text{input } 4 \\
\text{function } f & \quad \text{function } f & \quad \text{function } g & \quad \text{function } g \\
5 \quad \text{output} & \quad 6 \quad \text{output} & \quad 2 \quad \text{output} & \quad 3 \quad \text{output}
\end{align*}
\]

\[f(3) = 5 \quad f(4) = 6 \quad g(3) = 2 \quad g(4) = 3\]

Exercise 1.2.5  Give three possible domains for a function defined by the formula $g(X) = \frac{2}{X - 5}$.

\[
\{X \neq 5\}, \{X > 5\}, \{X < 5\}
\]

Exercise 1.2.6  Describe the everyday function examples you came up with in Exercise 1.2.1 on page 9 in “function name : domain → codomain” notation.

**Identification** : \{ID numbers\} $\rightarrow$ {names}

**US city population** : \{city\} $\rightarrow$ \{population size\}

Exercise 1.2.7  Suppose life is discovered on Mars. The Martians’ genetic code is remarkably similar to ours, but the RNA codon AUC is translated to serine 60% of the time and histidine 40% of the time. Is Martian gene expression a function?

Martian gene expression is not a function because the RNA codon, AUC, does not match to exactly one amino acid.
Exercise 1.2.8  As we saw earlier, a coffee shop menu is a function. Suppose that when you buy a drink, you have to pay 10% sales tax in addition to the price of the drink, so the total cost (price and tax) of a drink is 1.1 times the price on the menu.

a) Refer to Figure 1.11 on page 9. What is the total cost of a mocha? A latte?

b) Describe the process of finding the total cost in terms of function composition.

a) Mocha: $3.45 + 3.45(0.1) = 3.795 \sim \$3.80$

Latte: $3.15 + 3.15(0.1) = 3.465 \sim \$3.47$

b) There are two functions:

1. Menu: {Hot Beverages} \rightarrow {Prices}
   Which can be described by figure 1.11, where $\text{Menu}(B) = P$

2. Tax: {Prices} \rightarrow {Prices}
   Which can be described by $\text{Tax}(P) = 1.1P$

The process of finding the total cost can, therefore, be described as

$\text{Tax(\text{Menu}(B))} = \text{total cost}$