1. Let’s go through each of the bullet points one by one.

- Great - we’ll have \( F \) and \( A \) as state variables, and a perfect answer to the question will give correct change equations for \( F' \) and \( A' \).
- This provides a term \(+2.4\) in the expression for \( F' \).
- This provides a term \(-0.85A\) in the expression for \( A' \).
- The last part is a reaction:

\[
F + 2A \rightarrow 3A. 
\]

This gives rise to a new term in each of the expressions for \( F' \) and \( A' \):

\[
F' = \ldots + (-1) F + (+1) A \\
A' = \ldots + (+1) F + (+1) A \\
FA^2 \]

\( FA^2 \) is the term that measures (in some units) the frequency at which a molecule of \( F \) comes into contact with two molecules of \( A \).

We conclude that the change equations are given by:

\[
F' = 2.4 - 1.6FA^2, \\
A' = -0.85A + 1.6FA^2. 
\]

I would give full credit for just these equations, but what if you make a mistake somewhere? You are unlikely to receive partial credit unless you also attempt to you explain your thinking for each of the terms.

A good way to express at least the correct number of terms and their sign is with an inflow and outflow diagram. This is what the instructions recommend.
2. We have state variables \(J\), \(M\), and \(F\). We need to give change equations for \(J'\), \(M'\), and \(F'\). Let’s go through each of the bullet points one by one.

- \(J\) birth is cause by male-female interactions. We have learned that these are controlled by the quantity \(MF\). This provides a term \(+5.4MF\) in the expression for \(J'\).
- This provides a term \(-0.43J\) in the expression for \(J'\).
  
  It also provides a term \(-0.26M\) in the expression for \(M'\), and a \(-0.26F\) in the expression for \(F'\).
- We have to process the sentence: “the per-capita rate at which juveniles become adult males is proportional to the inverse of the adult male population, with a proportionality constant of \(0.11\)” This process provides an outflow for \(J\) and an inflow for \(M\). Per capita means “per juvenile”, so we will have terms that looks like \((...).J\). The bit inside the parentheses needs to deal with the clause: “is proportional to the inverse of the adult male population, with a proportionality constant of \(0.11\)” It is \(\frac{0.11}{M}\). 
  
  So we have a \(-\frac{0.11}{M}J\) term for \(J'\) and a \(+\frac{0.11}{M}J\) term for \(M'\).
- Similar to the last bullet point, we have a term \(-\frac{0.83}{M}M\) for \(M'\) and a \(+\frac{0.83}{M}M\) for \(F'\).

We conclude that the change equations are given by:

\[
J' = 5.4MF - 0.43J - \frac{0.11}{M}J, \\
M' = -0.26M + \frac{0.11}{M}J - \frac{0.83}{F}M, \\
F' = -0.26F + \frac{0.83}{F}M. 
\]

3. Based on owls preying on rabbits, and rabbits eating grass, we can draw part of an inflow and outflow diagram.

Rabbits as food inspire owls to reproduce \(\rightarrow\) Owls 

Grass as food inspires rabbits to reproduce \(\rightarrow\) Rabbits \(\rightarrow\) Rabbits are eaten by owls 

\(\rightarrow\) Grass is eaten by rabbits 

All of these inflows and outflows rely on an interaction. So these inflows and outflows should show up as terms involving \(XY\), \(YZ\), or \(XZ\). 

(a) \(X = \text{rabbits}, Y = \text{grass}, Z = \text{owls}\).

I figured this out by continuing with the thoughts above. Only the expression for \(X'\) involves a positive and negative interaction term. So \(X\) must be rabbits.

Only \(Y'\) involves a single negative interaction term.

Only \(Z'\) involves a single positive interaction term.
(b)  i. Grass being eaten by rabbits.
    ii. Soil can only support so much grass. 500 is the carrying capacity of the environment.
    iii. Owls die. This term assumes they die at a per capita rate of 0.1.
    iv. Owls reproduce at a per capita rate proportion to the amount of food available, that
        is, proportional to the rabbit population.
    v. Rabbits compete with each other for food. As a result of this competition, they die.

4. In the real question, numbers have been chosen so that your arrows are a reasonable size.
   We (Will Conley and I) suggest you look for patterns to make your drawing more efficient: in
   this example, the arrows coming from the X-axis have no X-component.

5. (a) No, it is not a function. My friend and I were both born on a Monday and we both live
    in Westwood. If it was a function, we’d have Me = f(Monday) = my friend!!

(b) Yes, the sorting hat chooses exactly one house for each student.

(c) Almost. It looks like the rule is int(the line described by $y = mx + b$) = b. However,
    vertical lines are not described by such a formula, and the vertical line $x = 1$ has no
    y-intercept.

(d) Define (take-home) : $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ by (take-home)(x) = 0.8x.
    The composition is (take-home) o (salary) : {employees} \rightarrow \mathbb{R}_+,
    and ((take-home) o (salary))(p) = p’s take-home salary.
6. There are many feedback loops in this system. I showed lots of them in the “review” on Friday. The real question won’t be so complicated.

The goal of this write up is to show how we might hope for your answer to look. I’ll give two correct answers.

\[
\begin{align*}
X' &= 3Y - Z \\
Y' &= X - 4Z \\
Z' &= X + 2Y \\
W' &= 2X + Y
\end{align*}
\]

Using the highlighted terms, we have the positive feedback system displayed below:

\[
\begin{align*}
X &\rightarrow + \rightarrow Y \\
&+ \\
X &\rightarrow + \rightarrow Y
\end{align*}
\]

\[
\begin{align*}
X' &= 3Y - Z \\
Y' &= X - 4Z \\
Z' &= X + 2Y \\
W' &= 2X + Y
\end{align*}
\]

Using the highlighted terms, we have the negative feedback system displayed below:

\[
\begin{align*}
X &\rightarrow + \rightarrow Y \\
- \rightarrow + \rightarrow Z \\
X &\rightarrow + \rightarrow Y
\end{align*}
\]

7. On the next few pages...

There was no way of knowing the time values from the original picture, so don’t worry about those details.
Wolves