In the book, they use the following notation for a chemical reaction:

$$2X + Y \xrightarrow{k} X + 3Y.$$ 

This means that a reaction takes:

- as input two \(X\) molecules and one \(Y\) molecule;
- as output one \(X\) molecule and three \(Y\) molecules;
- there is a proportionality constant \(k\) that will show up.

They note that we can use such notation to describe many other dynamical systems. For example, the Shark-Tuna model is the result of 3 “reactions”:

\[
\begin{align*}
S & \xrightarrow{d} 0, \\
T & \xrightarrow{b} 2T, \\
S + T & \xrightarrow{\beta} S + mS.
\end{align*}
\]

(The last formula disagrees with the one in the textbook on page 36. I can tell you my reasons for this in person. But it’ll certainly check out with what I’m about to show you.)

I like to add an extra decoration to these diagrams to indicate what we have to add to the left quantity to get the right quantity. The first example becomes:

$$2X + Y \xrightarrow{k}{(-1)X + (+2)Y} X + 3Y.$$ 

The Shark-Tuna model becomes:

\[
\begin{align*}
S & \xrightarrow{d}{(-1)S} 0, \\
T & \xrightarrow{b}{(+1)T} 2T, \\
S + T & \xrightarrow{\beta}{(+m)S + (-1)T} S + mS.
\end{align*}
\]
The number of interactions of two $X$ molecules with one $Y$ molecule is controlled by the quantity $X^2Y$. A reaction causes us to lose one $X$ and gain two $Y$s so we might expect:

\[
X' = -X^2Y, \\
Y' = 2X^2Y.
\]

This is almost correct. $k$ is the factor we need to fix it:

\[
X' = -kX^2Y, \\
Y' = 2kX^2Y.
\]

Similarly, $S \xrightarrow{d} (-1)S \rightarrow 0$ gives $S' = -dS$, $T \xrightarrow{b} (+1)T \rightarrow 2T$ gives $T' = bT$, and

\[
S + T \xrightarrow{\beta} (mS + (-1)T) \rightarrow S + mS
\]
gives $S' = m\beta ST$ and $T' = -\beta ST$. Of course, all these “reactions” happen at the same time, so we end up with the ST-model:

\[
S' = m\beta ST - dS, \\
T' = bT - \beta ST.
\]

Hopefully the following diagram highlights the general idea. I can say more in the review session.

![Diagram](image_url)