The wave function of a system should obey these three rules:

\[ -\frac{\hbar^2}{8\pi^2 m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \]

\[ \int_{-\infty}^{+\infty} P(x)dx = \int_{-\infty}^{+\infty} [\psi(x)]^2 dx = 1 \]

\[ \psi \to 0 \text{ as } x \to \pm \infty \]

For 1D particle in a box we have

\[ \psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \quad n = 1, 2, 3, \ldots \]

\[ E_n = \frac{n^2 \hbar^2}{8mL^2} \quad n = 1, 2, 3, \ldots \]

For 2D and 3D, particle-in-a-box, the potential energy is constant in all directions inside the box, so the motions in the x direction are independent of the motions in the y and z directions, and vice versa. The Schrödinger equation can be solved by the method of separation of variables. The wave function is the product of the wave functions for independent motion in each direction, and the energy is the sum of the energies for independent motion in each direction.

\[ E_{n_x n_y n_z} = \frac{\hbar^2}{8mL^2} \left[ n_x^2 + n_y^2 + n_z^2 \right] \]

\[ \begin{cases} n_x = 1, 2, 3, \ldots \\ n_y = 1, 2, 3, \ldots \\ n_z = 1, 2, 3, \ldots \end{cases} \]