Throughout today \( f \) is a function of \( x \).

Its graph is the set of points with

\[
y = f(x).
\]

**Defn:** The average rate of change of \( f \) (with respect to \( x \)) between two values \( x_0 \) and \( x_1 \) is

\[
\frac{\text{change in } f}{\text{change in } x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}.
\]
LET $\Delta X = X_1 - X_0$.
So $X_1 = X_0 + \Delta X$.

**THE AVERAGE RATE OF CHANGE OF**

**OF** $f(\text{WRT } X)$

**BETWEEN** $X_0$ AND $X_0 + \Delta X$

**IS**

$$\frac{f(X_0 + \Delta X) - f(X_0)}{\Delta X}$$

**DEFN:** **THE INSTANTANEOUS RATE OF CHANGE**

**OF** $f(\text{WRT } X)$ **AT** $X_0$

**IS**

$$\lim_{\Delta X \to 0} \frac{f(X_0 + \Delta X) - f(X_0)}{\Delta X}$$

$$= \lim_{\Delta X \to 0} \left( \text{AVERAGE RATE OF CHANGE OF } f \right)$$

**OF** $\text{(WRT } X)$ **BETWEEN** $X_0$ AND $\Delta X$.

**OFTEN WRITE** $f'(X_0)$ **FOR THIS VALUE**.

**SOMETIMES** $\frac{df}{dx} \bigg|_{x=X_0}$. 

$\square$
EXAMPLES: \( f(x) = x^2 \).

1) **CALCULATE THE AVERAGE RATE OF CHANGE** of \( f \) ** BETWEEN** \( x_0 = 5 \) ** AND** \( x_1 = 6 \).

2) **CALCULATE THE INSTANTANEOUS** \( R \) ** OF** \( C \) ** OF** \( f \) ** AT** \( x_0 = 5 \).

3) **CALCULATE THE INSTANTANEOUS** \( R \) ** OF** \( C \) ** AT ANY POINT** \( x_0 \).

1) \[
\frac{f(6) - f(5)}{6 - 5} = 6^2 - 5^2 = 11.
\]

2) \[
f'(5) = \lim_{\Delta x \to 0} \frac{f(5 + \Delta x) - f(5)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(5^2 + 2 \cdot 5 \cdot \Delta x + \Delta x^2) - 5^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{10 \Delta x + \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} 10 + \Delta x = 10.
\]
\[ \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \]

= SLOPE OF BLUE LINE IN SLIDES

As \( \Delta x \to 0 \),

THIS GETS CLOSER AND CLOSER

TO SLOPE OF TANGENT LINE.

\[ f'(x_0) = \text{SLOPE OF TANGENT LINE} \]

TO \( y = f(x) \)

AT \((x_0, f(x_0))\).
CLICKER Q:

A: No.
B: YES, AT 1 POINT
C: YES, AT 2 POINTS
D: YES, AT 3 POINTS.
\[
\frac{y - f(x_0)}{x - x_0} = \text{Slope of Tangent Line} = f'(x_0).
\]
EQUATION OF TANGENT LINE

To \( y = f(x) \) at \( (x_0, f(x_0)) \)

is

\[ y - f(x_0) = f'(x_0) \cdot (x - x_0). \]

EXAMPLE.

What's the tangent line to the graph of \( f(x) = x^2 \) at the point where \( x = 3 \)?