LAST TIME

INSTANTANEOUS \( R \) OF \( C \) OF \( f (\text{VAR X}) \) AT \( X_0 \).

\[
\lim_{{\Delta X \to 0}} \frac{f(X_0 + \Delta X) - f(X_0)}{\Delta X}
\]

\( f'(X_0) \).

EQU. OF TANGENT LINE TO \( y = f(X) \)
AT \( (X_0, f(X_0)) \) IS

\[
y - f(X_0) = f'(X_0) \cdot (X - X_0).
\]

DIFFERENTIATION RULES

\[
\frac{d}{dx} (x^n) = nx^{n-1}
\]

\[
\frac{d}{dx} (e^{kx}) = ke^{kx}
\]

\[
\frac{d}{dx} (\cos(x)) = -\sin(x)
\]

\[
\frac{d}{dx} (\sin(x)) = \cos(x).
\]
In our Calculus description, variable was $x$.

Could have functions of different variables. In particular, $t =$ time is common.

Let $x(t) = e^{bt}$.

Then $x'(t) = \frac{d}{dt}(e^{bt}) = b e^{bt} = b x(t)$.

So $x(t)$ solves equation $x' = bx$.

(There are other solutions. To pin down need to know initial conditions.)

Let $J(t) = \cos t$, $R(t) = -\sin t$.

Then $J'(t) = \frac{d}{dt}(\cos t) = -\sin t = R(t)$

And $R'(t) = \frac{d}{dt}(-\sin t)$

A rule we'll discuss shortly

So $J(t) = \cos t$ solves equations $J' = R$, $R' = -J$. 

Rushed last time so slower now...
OG RULES

\[
\frac{d}{dx} \left( c f(x) \right) = c \frac{d}{dx} (f(x))
\]

\[
\frac{d}{dx} \left( f(x) + g(x) \right) = \frac{d}{dx} (f(x)) + \frac{d}{dx} (g(x))
\]

(FANCY WAY OF SAYING THIS IS "DIFFERENTIATION IS A LINEAR FUNCTION")

EXAMPLE

LET \( f : \mathbb{R} \to \mathbb{R} \) BE DEFINED BY

\[
f(x) = 7x^5 - 3x^2.
\]

WHAT IS \( f'(x) \)?
A FUNCTION \( f(x) \) HAS A DERIVATIVE AT \( x_0 \) PROVIDED

\[
\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}
\]

EXISTS. IN THAT CASE

\[
f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}
\]

THIS EQUATION MEANS

"AS \( \Delta x \) GETS CLOSER TO 0, \( f(x_0 + \Delta x) - f(x_0) \)

\[
\frac{\Delta x}{\Delta x}
\]

GETS CLOSER TO THE NUMBER \( f'(x_0) \).\)
When $\Delta x$ is close to 0, we have

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x_0) \quad (1)$$

That's just resaying \((\text{average rate of change})\) approximates \((\text{instantaneous rate of change})\).

From (1), we get

$$f(x_0 + \Delta x) - f(x_0) = f'(x_0) \cdot \Delta x \quad (2)$$

From (2), we get

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0) \cdot \Delta x \quad (3)$$

(1)-(3) are all saying the same thing in slightly different ways.
Notice, if we talk about \( X(t) \) we get

\[
X(t_0 + \Delta t) \approx X(t_0) + X'(t_0) \cdot \Delta t
\]

From which we get Euler's method.

(2) and (3) are often described as "linear approximation".

For the most part, linear approx. is useful as a concept, e.g. it underpins Euler's method.

Let's see at least one example of a calculation we can do with it.

Q: What is \( \sqrt{101} \)?

No calculator allowed!
\[ \sqrt{101} \approx \sqrt{100} + \frac{1}{2\sqrt{100}} \]

\[ = 10.05 \]

\[ \sqrt{102} \approx 10.1 \]

\[ \gamma - 10 = \frac{1}{20} (x - 100) \]

\[ \gamma = -\sqrt{x} \]