1. In each part below, compute the derivative $X'(t)$.

(a) $X(t) = t^2e^t$

(b) $X(t) = e^{\sin(t)+t}$

(c) $X(t) = \ln t - 5t^3$

(d) $X(t) = \frac{\cos(t)}{7t+9}$
2. Let $X(t) = t^3 - t$. Compute the derivative $X'(t)$ using the difference quotient definition of $X'(t)$. (That is, compute $X'(t)$ by computing $\lim_{\Delta t \to 0} \frac{X(t + \Delta t) - X(t)}{\Delta t}$.)
3. Consider the Romeo and Juliet change equation

\[ R' = 2R - J, \]
\[ J' = -(0.3)R + J \]

with initial condition/initial Euler step

\[ R_0 = -1, \]
\[ J_0 = 1. \]

Compute the next three Euler steps, \((R_1, J_1)\), \((R_2, J_2)\), and \((R_3, J_3)\), taking \(\Delta t = 0.5\).
4. The mass of a tree, $M$ (kilograms) is related to its diameter, $D$ (centimeters) by an allometric equation of the form:

$$M(D) = aD^b$$

Taking $a = 3$ and $b = 2$, so that $M(D) = 3D^2$, answer the following questions:

(a) Compute $M'(2)$.

(b) Write down the equation that relates $\Delta M$ (a small change in $M$) to $\Delta D$ (a small change in $D$) near $D = 2$.

(c) Approximate the change in $M$ that results when $D$ is decreased from 2 to 1.7.
5. Let $P(t)$ denote the number of platypi in all of Tasmania at time $t$ (years), and assume that $P(t) = \frac{20t^2}{1+t^2}$.

(a) Compute $P(3)$, and then compute the tangent line to the graph of $P(t)$ at the point $(3, P(3))$.

(b) Using the tangent line you found in part (a), approximate the value of $P(5.5)$. 

Page 6
6. Consider the predator-prey dynamics of three interacting animal populations in the Sahara desert: rodents \((R)\), sand cats \((S)\), and wild dogs \((D)\). Wild dogs prey on sand cats, and sand cats prey on rodents.

(a) Based on the following assumptions, write down change equations for \(R\), \(S\), and \(D\) that describe how each variable changes with time:

- Rodents are born at a per capita rate \(a\).
- Rodents are eaten by sand cats at a per rodent rate proportional to the number of sand cats present, with proportionality constant \(b\).
- Sand cats are born at a rate proportional to the total rate at which rodents are eaten, with proportionality constant \(c\).
- The rate at which sand cats are eaten by wild dogs is represented with the mass action law, with constant \(d\).
- The rate at which wild dogs are born is proportional to the total rate at which sand cats are eaten, with proportionality constant \(g\).
- Wild dogs die at a constant per capita rate of \(h\).
(b) Scientists who study animal populations in the Sahara discover that the sand cat population is very small compared to the wild dog population. They decide to implement a wild dog “trap-and-neuter” program, which should reduce the wild dog birth rate. How would you modify the model to study the effect of this program, if it manages to reduce the birth rate to 65% of its current value?

(c) The program in part (b) is so successful that the sand cat population rebounds and actually exceeds a healthy size. The huge sand cat population causes the rodent population to dwindle, and the scientists then decide to implement a sand cat “declawing” program, which should reduce the rate at which rodents are eaten. How would you modify the model the study the effect of this program, if it manages to reduce this rate by half?
(scratch paper)