A study guide for Russell's first puzzle from "On Denoting"

Question 1: Does the conclusion follow from the premises by Leibniz's Law as the puzzle claims?

ANS: No.

Question 2: Why?

ANS: Once the description is eliminated, the second premise is not of the form A = B. So Leibniz's Law does not apply.

Question 3: Then why does it seem that so many inferences with exactly the same second premise are valid, for example, "I had dinner with the author of Waverley. Scott = the author of Waverley. ∴ I had dinner with Scott."

ANS: Arguments of this form are valid (though not by a single Leibniz's Law step) whenever the description in the first premise has primary scope.

Question 4: How does that work? Such an argument would have the form,
\[ \exists x[(Wx \land \neg \exists y(y \neq x \land Wy)) \land Dx]. \exists x[(Wx \land \neg \exists y(y \neq x \land Wy)) \land a = x] \therefore Da \]

How could we give a derivation for this argument?

ANS: EI the two premises to i and j. QN and NC to get \( \forall y(Wy \rightarrow y = i) \). UI and MP to get \( j = i \). Transitivity of identity yields \( a = i \), which by LL yields Da.

Question 5: So, if in the first premise of the argument in question - "George IV wished to know whether Scott was the author of Waverley" - we give the description primary scope, the argument to the false conclusion would go through?

ANS: Yes.

Question 6: What, intuitively, does the first premise say when we give the description primary scope? Russell says something about seeing a man in the distance.

ANS: It is as if George IV saw someone in the distance and wonder if Scott were that man. In reporting this event, the reporter, who is aware of whom George IV was seeing, characterizes that person as a person who uniquely wrote Waverley.
\[ \exists x[(Wx \land \neg \exists y(y \neq x \land Wy)) \land George IV wished to know whether Scott = x] \]

Question 7: OK. Does this describe what happened at the dinner?

ANS: No. George IV recognized Scott as soon as he saw him. He wasn't wondering if the man he was looking at was Scott. He was wondering if the man he was looking at was the author of Waverley (that is, if the man he was looking at was a unique author of Waverley).
Question 8: OK. I see that to accurately describe what happened at dinner the phrase indicating George IV’s state of mind must have scope over the definite description "the author of Waverley", because what George IV was wondering about had to do with authorship of the novel Waverley. This is what Russell called giving the definite description "secondary scope". How would Russell eliminate the definite description when it has secondary scope, as it must to accurately describe what happened at dinner?

ANS: George IV wished to know whether $\exists x[(Wx \land \neg \exists y(y \neq x \land Wy)) \land Scott = x]$

Question 9: How do we know we can't derive the conclusion using this version of the first premise?

ANS: There are no rules of pure logic that allow one to operate on the internal parts of a formula. Even the rule of the Interchange of Equivalents (IE) is a derived rule, and it would not be derivable if the language contained sentential operators like, "George IV wished to know whether". Were we to consider adding special rules of ‘epistemic logic’ governing this operator, there is not much that we could plausibly add. If what he wished to know logically implies something else, it doesn't follow that he wished to know that something else. For example, he may have wished to know whether there are flying horses. It doesn't follow that he wished to know whether there are horses. Similarly if something else logically implies what he wished to know, it also doesn't follow that he wished to know that something else. For example he may have wished to know whether there are black swans. It doesn't follow that he wished to know whether there are reptilian black swans.

Question 10: So is this Russell's ultimate solution to his first puzzle: if the description has secondary scope, then the first premise is true in the context of the dinner but the conclusion doesn't follow, and if the description has primary scope, then the conclusion does follow but the first premise is not true in the context of the dinner.

ANS: Yes.

Question 11: But suppose that after the dinner, George IV was leaving in his carriage and saw a man in the distance whom he wasn't sure he recognized. So he asked his cabbie, "Is that Scott?". Wouldn't this scenario make the first premise - with the description being given primary scope - true, according to Russell?

ANS: Yes.

Question 12: If so, since the second premise remains true in any case, doesn't it follow by the argument given in answer to question 4 that, as Russell would put it, the first gentleman of Europe has an interest in the law of identity?

ANS: It might seem so.
Question 13: Doesn't Russell need to block this argument also?

ANS: To the best of my knowledge Russell never took up this question and so never faced the problem that this scenario poses.

Question 14: Even if he didn't say anything explicitly about such an argument how might he have approached it?

ANS: The first thing to note is that Russell speaks of the description having a primary occurrence "in the proposition considered". We have taken this to refer to the first premise of his argument, and I think this is correct. However, if we give the definite description in the first premise a primary occurrence, then following the "verbal substitution" in that premise, whatever replaces the definite description should also have a primary occurrence. We can see that this is probably what Russell intended by replacing the description with another definite description (for example "the author of Marmion") rather than with the name "Scott". In this case the argument would become:

George IV wished to know whether the author of Marmion was the author of Waverley

The author of Marmion = the author of Waverley

Therefore

George IV wished to know whether the author of Marmion was the author of Marmion

we can give a derivation for this argument, just as Russell claimed, provided that "the author of Waverley" in the first premise and the second occurrence of "the author of Marmion" in the conclusion both have primary occurrences. (I'll leave it as a homework problem to carry out the derivation.) Otherwise, the derivation will not go through. In fairness to Russell, this is probably what he had in mind. It is a certain position in the first premise whose occupant we treat as having a primary occurrence, and whatever replaces the occupant of that position should also have primary occurrence.

But we can not yet absolve Russell of all fault. He speaks explicitly of verbally substituting "Scott" for "the author of Waverley", but he has not given us any theory whatsoever of scope as it applies to names like "Scott". If we are to avoid the conclusion that the first gentleman of Europe has an interest in the law of identity, we will have to find a way to give primary scope to the name "Scott" when it replaces the description in the conclusion.

The issue of scope, what Russell calls primary and secondary occurrences of the definite description, is critical to Russell's resolution of both the first and second puzzles. In the case of the second puzzle it gives him a way of dealing with improper, or empty, descriptions. In the case of the first puzzle, the description is proper, but scope is used to
disambiguate what Russell hears as an ambiguous sentence (the first premise of the argument). Now in addition to definite descriptions that are empty there are also empty names. An example that Russell liked to use was "Apollo". Russell treated such names as abbreviating definite descriptions, and so gave them scope. Another way to give names scope would be by replacing each original occurrence of a name by a ‘circular’ definite description such as “The x x=Scott”, “The x x= Apollo”, and so on. This again would automatically give them scope. (This proposal may be equivalent to the proposal below.) Note that the addition of empty names, like "Apollo", to the language of logic requires some adjustments in the quantifier laws, but we know how to do that. The addition of propositional attitude operators, like "George IV wished to know whether", also requires some adjustments to logical laws such as Leibniz's Law.

But even without treating an empty name as abbreviating a definite description, it seems that we can replicate Russell's second puzzle. And, if we give names scope, so that we can speak of primary and secondary occurrences of the name, we should be able to replicate Russell's solution.

Here is a version of the second puzzle, but for names instead of definite descriptions: The law of bivalence tells us that for any sentence either it or its negation is true. But the sentence "Apollo is bald" is not true because if we line up all of the bald things, we will not find Apollo among them. On the other hand the sentence "Apollo is not bald" is also not true because if we line up all of the not-bald things, we will not find Apollo among them either. Could the Sun God wear a wig?

To make the appropriate distinction between primary and secondary occurrences of the name "Apollo" we need a way to represent the case in which the name has scope over the negation. In English we might formulate the difference as that between "Apollo is an individual who is not bald" and "it is not the case that Apollo is bald". The simplest way to symbolize these is as,

$$\exists x [a = x \land \neg B x] \quad \text{and} \quad \neg B a.$$

Note that these two are not equivalent since the second will not imply the first in a logic that allows for empty names. (In such a logic we would not be permitted to existentially generalize on an empty name.)

If we accept Russell's claim that simple sentences with empty subjects, sentences such as "Apollo is bald", are plainly false (p 484 in OD), then we will take "Ba" to be false, and hence "~Ba" to be true. This sentence is the true logical negation of "Ba" ("Apollo is bald"). On the other hand the symbolization that gives "Apollo" primary scope is, as the argument about lining up the not-bald things shows, false. So there is a reading of "Apollo is not bald" that is true (this reading is the true logical negation of "Apollo is bald"), and there is a reading of "Apollo is not bald" that is false (just as "Apollo is bald" is).
Now if we apply this idea that a name can have either a primary or a secondary occurrence to the first puzzle, using the scenario in which George IV sees Scott at a distance and does not recognize him ("Is Scott that man?" he asks), we will obtain,

\[ \exists x [a = x \land \text{George IV wishes to know whether } a = x] \]

This is one reading of "George IV wished to know whether Scott is Scott", but it is the one that gives the second "Scott" a primary occurrence, not the one in which George IV is exhibiting an interest in the law of identity. The reading on which George IV is exhibiting an interest in the law of identity must give both occurrences of "Scott" secondary occurrences. On this reading George IV is entertaining a thought that involves two appearances of the name "Scott"; the other reading is the one in which George IV is entertaining a thought that involves one appearance of the name "Scott" and perhaps one appearance of a representation of the man he is seeing in the distance. The secondary occurrence reading should not follow from the primary occurrence reading (the one symbolize the above), and so the rule of Leibniz's Law would have to be restricted when applied across propositional attitude operators like "George IV wishes to know whether" in order to prevent such a derivation from going through. Intuitively, such a derivation should not go through, so the restriction on Leibniz's Law seems justified.

To sum up. If the second premise were an identity between two descriptions, for example a "the author of Marmion = the author of Waverley", and the first premise were "George IV wished to know whether the author of Marmion = the author of Waverley", then if we give "the author of Waverley" a primary occurrence in the first premise, we can make what is verbally the substitution of "the author of Marmion" for "the author of Waverley" in the conclusion to obtain "George IV wished to know whether the author of Marmion = the author of Marmion", provided that we also give the replacing expression (the second "the author of Marmion") a primary occurrence in the conclusion. When I say we can make this substitution, what I mean is that we can carry out the derivation. Similarly, when the second premise is an identity between a name and a description, as it is in Russell's actual example, we should again be able to carry out the derivation provided that we give the replacing name a primary occurrence in the conclusion. And this of course requires that we have a notion of primary and secondary occurrences for names. Russell did not have this, at least he did not have it without resorting to the doctrine that all names abbreviated definite descriptions, and this lack seems a defect in Russell's theory.

[It should be noted that although the foregoing may seem to a modern eye like the natural way of resolving the puzzle about George IV seeing Scott in the distance, we have wandered quite far from Russell's own epistemological ideas, especially those about acquaintance. According to those ideas, if the individual stating the argument were acquainted with Scott, the name “Scott” would function like a genuine proper name, allowing us to move it from primary scope to secondary scope. But then]
the conclusion would state that George IV was interested in an instance of the law of identity.]

Consideration of the names "Scott" and "Apollo" highlights the importance of two of Russell's foundational ideas. First, that all singular terms (both definite descriptions and names) must be given scope. And second, that a sentence or sub-sentence with an empty singular term (either a definite description or a name) in subject position, i.e. having primary scope in the sentence or sub-sentence, should be treated as plainly false. These two ideas alone may yield a theory equivalent to Russell's¹ but without requiring the elimination of definite descriptions and without abandoning Russell's original idea that a definite description expresses a denoting concept that denotes an individual (or, if empty, denotes nothing). Some tinkering would have to be done with Russell's original conception of the role that denoting concepts played in propositions in order to account for the fact that these denoting concepts would have to have scope. But propositions containing denoting concepts, with or without scope, would certainly accord much better with Russell's epistemological ideas concerning knowledge by description.

DK

¹ Here I am thinking of an equivalence like that I mention in connection with “Russell's theorem's of descriptions” during the discussion with Yourgrau of my paper "What is Russell's theory of descriptions?". (The discussion is appended to the end of the paper.)