well as 9. And so it is nonsense to say neutrally that there is something, $x$, that necessarily exceeds 7. Church countered that my argument worked only for things like numbers, bodies, classes, that we could specify in contingently coincident ways: thus 9 is what succeeds 8, and is what numbers the planets, and these two specifications only contingently coincide. If we limit our objects to intensions, Church urged, this will not happen.

Now on this latter point Church was wrong. I have been slow to see it, but the proof is simple. Anything $x$, even an intension, is specifiable in contingently coincident ways if specifiable at all. For suppose $x$ is determined uniquely by the condition ‘$p, x’’. Then it is also determined uniquely by the conjunctive condition ‘$p \cdot p, x’’ where ‘$p’’ is any truth, however irrelevant. Take ‘$p’’ as an arbitrary truth not implied by ‘$p, x’’, and these two specifications of $x$ are seen to be contingently coincident: ‘$p, x’’ and ‘$p, p, x’’.

Contrary to what Church thought, therefore, my 1943 strictures were cogent against quantification over any sorts of objects if cogent at all; nothing is gained by limiting the universe to intensions. The only course open to the champion of quantified modal logic is to meet my strictures head on: to argue in the case of 9 and the number of the planets that this number is, of itself and independently of mode of specification, something that necessarily, not contingently, exceeds 7. This means adopting a frankly inequalitarian attitude toward the various ways of specifying the number. One of the determining traits, the succeeding of 8, is counted as a necessary trait of the number. So are any traits that follow from that one, notably the exceeding of 7. Other uniquely determining traits of the number, notably its numbering the planets, are discounted as contingent traits of the number and held not to belie the fact that the number does still necessarily exceed 7.

This is how essentialism comes in: the invidious distinction between some traits of an object as essential to it (by whatever name) and other traits of it as accidental. I do not say that such essentialism, however uncongenial to me, should be uncongenial to the champion of quantified modal logic. On the contrary, it should be every bit as congenial as quantified modal logic itself.²

² For more in the vein of these last few paragraphs see my From a Logical Point of View, 2d ed., pp. 148-157.

**Quantifiers and Propositional Attitudes**

The incorrectness of rendering ‘Ctesias is hunting unicorns’ in the fashion:

$$(\exists x)(x \text{ is a unicorn } \land \text{ Ctesias is hunting } x)$$

is conveniently attested by the non-existence of unicorns, but is not due simply to that zoological lacuna. It would be equally incorrect to render ‘Ernest is hunting lions’ as:

$$(\exists x)(x \text{ is a lion } \land \text{ Ernest is hunting } x)$$

where Ernest is a sportsman in Africa. The force of (1) is rather that there is some individual lion (or several) which Ernest is hunting; stray circus property, for example.

The contrast recurs in ‘I want a sloop’. The version:

$$(\exists x)(x \text{ is a sloop } \land \text{ I want } x)$$

is suitable insofar only as there may be said to be a certain sloop that I want. If what I seek is mere relief from slooplessness, then (2) gives the wrong idea.

The contrast is that between what may be called the relational sense of lion-hunting or sloop-wanting, viz., (1)–(2), and the

This paper appeared in the Journal of Philosophy (Volume 53, 1956), summing up some points which I had made in lectures at Harvard and Oxford from 1952 onward. It is reprinted here minus fifteen lines.
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likely or notional sense. Appreciation of the difference is evinced in Latin and Romance languages by a distinction of mood in subordinate clauses; thus 'Procuro un perro que habla' has the relational sense:

$\exists x (x$ is a dog. $x$ talks. I seek $x$)

as against the notional 'Procuro un perro que hable':

I strive that $(\exists x)(x$ is a dog. $x$ talks. I find $x$).

Pending considerations to the contrary in later pages, we may represent the contrast strikingly in terms of permutations of components. Thus (1) and (2) may be expanded—with some violence to both logic and grammar—as follows:

(3) $(\exists x)(x$ is a lion. Ernest strives that Ernest finds $x$),

(4) $(\exists x)(x$ is a sloop. I wish that I have $x$),

whereas 'Ernest is hunting lions' and 'I want a sloop' in their notional senses may be rendered rather thus:

(5) Ernest strives that $(\exists x)(x$ is a lion. Ernest finds $x$),

(6) I wish that $(\exists x)(x$ is a sloop. I have $x$).

The contrasting versions (3)–(6) have been wrought by so paraphrasing 'hunt' and 'want' as to uncover the locutions 'strive that' and 'wish that', expressive of what Russell has called propositional attitudes. Now of all examples of propositional attitudes, the first and foremost is belief; and, true to form, this example can be used to point up the contrast between relational and notional senses still better than (3)–(6) do. Consider the relational and notional senses of believing in spies:

(7) $(\exists x)(Ralph believes that $x$ is a spy),

(8) Ralph believes that $(\exists x)(x$ is a spy).

Both may perhaps be ambiguously phrased as 'Ralph believes that someone is a spy', but they may be unambiguously phrased respectively as 'There is someone whom Ralph believes to be a spy' and 'Ralph believes there are spies'. The difference is vast; indeed, if Ralph is like most of us, (8) is true and (7) false.

In moving over to propositional attitudes, as we did in (3)–(6), we gain not only the graphic structural contrast between (3)–(4) and (5)–(6) but also a certain generality. For we can now multiply examples of striving and wishing, unrelated to hunting and wanting. Thus we get the relational and notional senses of wishing for a president:

(9) $(\exists x)(Witold wishes that $x$ is president),

(10) Witold wishes that $(\exists x)(x$ is president).

According to (9), Witold has his candidate; according to (10) he merely wishes the appropriate form of government were in force. Also we open other propositional attitudes to similar consideration—as witness (7)–(8).

However, the suggested formulations of the relational senses—viz., (3), (4), (7), and (9)—all involve quantifying into a propositional-attitude idiom from outside. This is a dubious business, as may be seen from the following example.

There is a certain man in a brown hat whom Ralph has glimpsed several times under questionable circumstances on which we need not enter here; suffice it to say that Ralph suspects he is a spy. Also there is a gray-haired man, vaguely known to Ralph as rather a pillar of the community, whom Ralph is not aware of having seen except once at the beach. Now Ralph does not know it, but the men are one and the same. Can we say of this man (Bernard J. Ortcutt, to give him a name) that Ralph believes him to be a spy? If so, we find ourselves accepting a conjunction of the type:

(11) $w$ sincerely denies '...'. $w$ believes that ...

as true, with one and the same sentence in both blanks. For, Ralph is ready enough to say, in all sincerity, 'Bernard J. Ortcutt is no spy'. If, on the other hand, with a view to disallowing situations of the type (11), we rule simultaneously that

(12) Ralph believes that the man in the brown hat is a spy,

(13) Ralph does not believe that the man seen at the beach is a spy,

then we cease to affirm any relationship between Ralph and any man at all. Both of the component 'that'-clauses are indeed about the man Ortcutt; but the 'that' must be viewed in (12) and (13) as sealing those clauses off, thereby rendering (12) and (13) compatible because not, as wholes, about Ortcutt at all. It then
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becomes improper to quantify as in (7); ‘believes that’ becomes, in a word, referentially opaque.1

No question arises over (8); it exhibits only a quantification within the ‘believes that’ context, not a quantification into it. What goes by the board, when we rule (12) and (13) both true, is just (7). Yet we are scarcely prepared to sacrifice the relational construction ‘There is someone whom Ralph believes to be a spy’, which (7) as against (8) was supposed to reproduce.

The obvious next move is to try to make the best of our dilemma by distinguishing two senses of belief: belief1, which disallows (11), and belief2, which tolerates (11) but makes sense of (7). For belief1, accordingly, we sustain (12)-(13) and ban (7) as nonsense. For belief2, on the other hand, we sustain (7); and for this sense of belief we must reject (13) and acquiesce in the conclusion that Ralph believes that the man at the beach is a spy even though he also believes2 (and believes1) that the man at the beach is not a spy.

II

But there is a more suggestive treatment. Beginning with a single sense of belief, viz., belief above, let us think of this at first as a relation between the believer and a certain intension, named by the ‘that’-clause. Intensions are creatures of darkness, and I shall rejoice with the reader when they are exorcised, but first I want to make certain points with the help of them. Now intensions named thus by ‘that’-clauses, without free variables, I shall speak of more specifically as intensions of degree 0, or propositions. In addition I shall (for the moment) recognize intensions of degree 1, or attributes. These are to be named by prefixing a variable to a sentence in which it occurs free; thus $x$ ($x$ is a spy) is spyhood. Similarly we may specify intensions of higher degrees by prefixing multiple variables.

Now just as we have recognized a dyadic relation of belief between a believer and a proposition, thus:

(14) Ralph believes that Ortcutt is a spy,

we may recognize also a triadic relation of belief among a believer, an object, and an attribute, thus:

(15) Ralph believes $x$ ($x$ is a spy) of Ortcutt.

so we may recognize also a triadic relation of belief among a believer, an object, and an attribute, thus:

(15) Ralph believes $z$ ($z$ is a spy) of Ortcutt.

For reasons which will appear, this is to be viewed not as dyadic belief between Ralph and the proposition that Ortcutt has $z$ ($z$ is a spy), but rather as an irreducibly triadic relation among the three things Ralph, $z$ ($z$ is a spy), and Ortcutt. Similarly there is tetradic belief:

(16) Tom believes $yz$ ($y$ denounced $z$) of Cicero and Catiline,

and so on.

Now we can clap on a hard and fast rule against quantifying into propositional-attitude idioms; but we give it the form now of a rule against quantifying into names of intensions. Thus, though (7) as it stands becomes unallowable, we can meet the needs which prompted (7) by quantifying rather into the triadic belief construction, thus:

(17) $(\exists z)(\text{Ralph believes } z (z \text{ is a spy}) \text{ of } z)$.

Here then, in place of (7), is our new way of saying that there is someone whom Ralph believes to be a spy.

Belief1 was belief so construed that a proposition might be believed when an object was specified in it in one way, and yet not believed when the same object was specified in another way; witness (12)-(13). Hereafter we can adhere uniformly to this narrow sense of belief, both for the dyadic case and for triadic and higher; in each case the term which names the intension (whether proposition or attribute or intension of higher degree) is to be looked on as referentially opaque.

The situation (11) is thus excluded. At the same time the effect of belief2 can be gained, simply by ascending from dyadic to triadic belief as in (15). For (15) does relate the men Ralph and Ortcutt precisely as belief2 was intended to do. (15) does remain true of Ortcutt under any designation; and hence the legitimacy of (17).

Similarly, whereas from:

Tom believes that Cicero denounced Catiline

we cannot conclude:

Tom believes that Tully denounced Catiline,
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on the other hand we can conclude from:

Tom believes $y(y$ denounced Catiline) of Cicero

that

Tom believes $y(y$ denounced Catiline) of Tully,

and also that

(18) $(\exists z)(\text{Tom believes } y(y$ denounced Catiline) of $z)$.

From (16), similarly, we may infer that

(19) $(\exists w)(\exists z)(\text{Tom believes } yz(y$ denounced $z)$ of $w$ and $z)$.

Such quantifications as:

$(\exists z)(\text{Tom believes that } x$ denounced Catiline),

$(\exists z)(\text{Tom believes } y(y$ denounced $x)$ of Cicero)

still count as nonsense, along with (7); but such legitimate purposes as these might have served are served by (17)–(19) and the like. Our names of intensions, and these only, are what count as referentially opaque.

Let us sum up our findings concerning the seven numbered statements about Ralph. (7) is now counted as nonsense, (8) as true, (12)–(13) as true, (14) as false, and (15) and (17) as true. Another that is true is:

(20) Ralph believes that the man seen at the beach is not a spy, which of course must not be confused with (13).

The kind of exportation which leads from (14) to (15) should doubtless be viewed in general as implicative. Under the terms of our illustrative story, (14) happens to be false; but (20) is true, and it leads by exportation to:

(21) Ralph believes $z(z$ is not a spy) of the man seen at the beach.

The man at the beach, hence Ortcutt, does not receive reference in (20), because of referential opacity; but he does in (21), so we may conclude from (21) that

(22) Ralph believes $z(z$ is not a spy) of Ortcutt.

Thus (15) and (22) both count as true. This is not, however, to charge Ralph with contradictory beliefs. Such a charge might reasonably be read into:

(23) Ralph believes $z(x$ is a spy . $x$ is not a spy) of Ortcutt;

but this merely goes to show that it is undesirable to look upon (18) and (22) as implying (23).

It hardly needs be said that the barbarous usage illustrated in (15)–(19) and (21)–(23) is not urged as a practical reform. It is put forward by way of straightening out a theoretical difficulty, which, summed up, was as follows: Belief contexts are referentially opaque; therefore it is prima facie meaningless to quantify into them; how then to provide for those indispensable relational statements of belief, like 'There is someone whom Ralph believes to be a spy?'

Let it not be supposed that the theory which we have been examining is just a matter of allowing unbridled quantification into belief contexts after all, with a legalistic change of notation. On the contrary, the crucial choice recurs at each point: quantify if you will, but pay the price of accepting near-contraries like (15) and (22) at each point at which you choose to quantify. In other words: distinguish as you please between referential and non-referential positions, but keep track, so as to treat each kind appropriately. The notation of intensions, of degree one and higher, is in effect a device for inking in a boundary between referential and non-referential occurrences of terms.

III

Striving and wishing, like believing, are propositional attitudes and referentially opaque. (3) and (4) are objectionable in the same way as (7), and our recent treatment of belief can be repeated for these propositional attitudes. Thus, just as (7) gave way to (17), so (3) and (4) give way to:

(24) $(\exists x)(x$ is a lion . Ernest strives $z(\text{Ernest finds } z)$ of $x)$,

(25) $(\exists x)(x$ is a sloop . I wish $z(\text{I have } z)$ of $x)$,

a certain breach of idiom being allowed for the sake of analogy in the case of 'strives'.

These examples came from a study of hunting and wanting. Observing in (3)–(4) the quantification into opaque contexts, then, we might have retreated to (1)–(2) and forborne to
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paraphrase them into terms of striving and wishing. For (1)-(2) were quite straightforward renderings of lion-hunting and sloop-wanting in their relational senses; it was only the notional senses that really needed the breakdown into terms of striving and wishing, (5)-(6).

Actually, though, it would be myopic to leave the relational senses of lion-hunting and sloop-wanting at the unanalyzed stage (1)-(2). For, whether or not we choose to put these over into terms of wishing and striving, there are other relational cases of wishing and striving which require our consideration anyway—as witness (9). The untenable formulations (3)-(4) may indeed be either corrected as (24)-(25) or condensed back into (1)-(2); on the other hand we have no choice but to correct the untenable (9) on the pattern of (24)-(25), viz., as:

\[(\exists x)(\text{Witold wishes } y(y \text{ is president}) \text{ of } x)\].

The untenable versions (3)-(4) and (9) all had to do with wishing and striving in the relational sense. We see in contrast that (5)-(6) and (10), on the notional side of wishing and striving, are innocent of any illicit quantification into opaque contexts from outside. But now notice that exactly the same trouble begins also on the notional side, as soon as we try to say not just that Ernest hunts lions and I want a sloop, but that someone hunts lions or wants a sloop. This move carries us, ostensibly, from (5)-(6) to:

(26) \[(3w)(w \text{ strives that } (3x)(x \text{ is a lion} \cdot w \text{ finds } x))\],

(27) \[(3w)(w \text{ wishes that } (3x)(x \text{ is a sloop} \cdot w \text{ has } x))\],

and these do quantify unallowably into opaque contexts.

We know how, with help of the attribute apparatus, to put (26)-(27) in order; the pattern, indeed, is substantially before us in (24)-(25). Admissible versions are:

(3w)(w strives \[y(y \text{ finds a lion}) \text{ of } w\]),

(3w)(w wishes \[y(y \text{ has a sloop}) \text{ of } w\]),

or briefly:

(28) \[(3w)(w \text{ strives } y(y \text{ finds a lion}) \text{ of } w)\],

(29) \[(3w)(w \text{ wishes } y(y \text{ has a sloop}) \text{ of } w)\].

Such quantification of the subject of the propositional attitude can of course occur in belief as well; and, if the subject is mentioned in the belief itself, the above pattern is the one to use. Thus 'Someone believes he is Napoleon' must be rendered:

\[(3w)(w \text{ believes } y(y = \text{Napoleon}) \text{ of } w)\].

For concreteness I have been discussing belief primarily, and two other propositional attitudes secondarily: striving and wishing. The treatment is, we see, closely parallel for the three; and it will pretty evidently carry over to other propositional attitudes as well—e.g., hope, fear, surprise. In all cases my concern is, of course, with a special technical aspect of the propositional attitudes: the problem of quantifying in.

There are good reasons for being discontent with an analysis that leaves us with propositions, attributes, and the rest of the intensions. Intensions are less economical than extensions (truth values, classes, relations), in that they are more narrowly individuated. The principle of their individuation, moreover, is obscure.

Commonly logical equivalence is adopted as the principle of individuation of intensions. More explicitly: if \(S\) and \(S'\) are any two sentences with \(n \geq 0\) free variables, the same in each, then the respective intensions which we name by putting the \(n\) variables (or 'that', if \(n = 0\)) before \(S\) and \(S'\) shall be one and the same intension if and only if \(S\) and \(S'\) are logically equivalent. But the relevant concept of logical equivalence raises serious questions in turn.² The intensions are at best a pretty obscure lot.

Yet it is evident enough that we cannot, in the foregoing treatment of propositional attitudes, drop the intensions in favor of the corresponding extensions. Thus, to take a trivial example, consider 'w is hunting unicorns'. On the analogy of (28), it becomes:

\(w \text{ strives } y(y \text{ finds a unicorn}) \text{ of } w\).

² See my "Two dogmas"; also "Carnap and logical truth," which is Essay 10 above.
Correspondingly for the hunting of griffins. Hence, if anyone \( w \) is to hunt unicorns without hunting griffins, the attributes

\[
\begin{align*}
y(y \text{ finds a unicorn}), \\
y(y \text{ finds a griffin})
\end{align*}
\]

must be distinct. But the corresponding classes are identical, being empty. So it is indeed the attributes, and not the classes, that were needed in our formulation. The same moral could be drawn, though less briefly, without appeal to empty cases.

But there is a way of dodging the intensions which merits serious consideration. Instead of speaking of intensions we can speak of sentences, naming these by quotation. Instead of:

\( w \) believes that . . .

we may say:

\( w \) believes-true ‘. . .’.

Instead of:

\( w \) believes \( y(\ldots y\ldots) \) of \( x \)

we may say:

\( w \) believes ‘. . . y . . .’ satisfied by \( x \).

The words ‘believes satisfied by’ here, like ‘believes of’ before, would be viewed as an irreducibly triadic predicate. A similar shift can be made in the case of the other propositional attitudes, of course, and in the tetradic and higher cases.

This semantical reformulation is not, of course, intended to suggest that the subject of the propositional attitude speaks the language of the quotation, or any language. We may treat a mouse’s fear of a cat as his fearing true a certain English sentence. This is unnatural without being therefore wrong. It is a little like describing a prehistoric ocean current as clockwise.

Now, where, and on what grounds to draw a boundary between those who believe or wish or strive that \( p \), and those who do not quite believe or wish or strive that \( p \), is undeniably a vague and obscure affair. However, if anyone does approve of speaking of belief of a proposition at all and of speaking of a proposition in turn as meant by a sentence, then certainly he cannot object to our semantical reformulation ‘\( w \) believes-true \( S \)’ on any special grounds of obscurity; for, ‘\( w \) believes-true \( S \)’ is explicitly defin-
and clearly (34) does not provide enough information to enable a German ignorant of English to infer (35).

The same reasoning can be used to show that 'There are unicorns' is not strictly or analytically equivalent to:

'There are unicorns' is true in English.

Nor, indeed, was Tarski's truth paradigm intended to assert analytic equivalence. Similarly, then, for (32) in relation to 'w believes that . . . '; a systematic agreement in truth value can be claimed, and no more. This limitation will prove of little moment to persons who share my skepticism about analyticity.

What I find more disturbing about the semantical versions, such as (32), is the need of dragging in the language concept at all. What is a language? What degree of fixity is supposed? When do we have one language and not two? The propositional attitudes are dim affairs to begin with, and it is a pity to have to add obscurity to obscurity by bringing in language variables too. Only let it not be supposed that any clarity is gained by restituting the intensions.

The characterization of axioms as implicit definitions can be found as far back as 1818, in Gergonne, and it was still vigorous thirty years ago. What is exasperating about the doctrine is its facility, or cheapness, as a way of endowing statements with the security of analytic truths without ever having to show that they follow from definitions properly so called, definitions with eliminable definienda.

Russell gave the doctrine its due, I felt, though he did not mention it by name, when he wrote in 1919 that “the method of ‘postulating’ what we want has many advantages; they are the same as the advantages of theft over honest toil.”

I am shocked now to find that the view of axioms as implicit definitions can be defended, and with a literalness that its own proponents can scarcely have thought possible. To discharge this somber duty is the purpose of the present note.

Briefly, the point is that there is a mechanical routine whereby, given an assortment of interpreted undefined predicates 'F₁', 'F₂', . . . , 'Fₙ' governed by a true axiom or a finite list of such, we can switch to a new and equally economical set of undefined predicates and define 'F₁', . . . , 'Fₙ' in terms of them, plus auxiliary arithmetical notations, in such a way that the old axioms

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