Physics - 5A

Week - 2
A projectile is an object that moves through the air under the influence of gravity and nothing else.

The path of the motion is a parabola.

The motion consists of two pieces:
1. Vertical motion with free-fall acceleration, \(a_y = -g\).
2. Horizontal motion with constant velocity.

Kinematic equations:

\[
\begin{align*}
x_f &= x_i + (v_x)_i \Delta t \\
(v_x)_f &= (v_x)_i = \text{constant} \\
y_f &= y_i + (v_y)_i \Delta t - \frac{1}{2} g(\Delta t)^2 \\
(v_y)_f &= (v_y)_i - g \Delta t
\end{align*}
\]
Circular Motion

For an object moving in a circle at a constant speed:

- The period $T$ is the time for one rotation.
- The frequency $f = 1/T$ is the number of revolutions per second.
- The velocity is tangent to the circular path.
- The acceleration points toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$
Vectors and Components

A vector can be decomposed into x- and y-components.

Magnitude \( A = \sqrt{A_x^2 + A_y^2} \)

Direction of \( \vec{A} \)

\[ \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \]

The magnitude and direction of a vector can be expressed in terms of its components.

The sign of the components depends on the direction of the vector:
The Acceleration Vector

We define the acceleration vector as

\[ \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t} \]

We find the acceleration vector on a motion diagram as follows:

- Dots show positions at equal time intervals.
- Velocity vectors go dot to dot.
- The acceleration vector points in the direction of \( \Delta \vec{v} \).
- The difference in the velocity vectors is found by adding the negative of \( \vec{v}_i \) to \( \vec{v}_f \).
Relative motion

Velocities can be expressed relative to an observer. We can add relative velocities to convert to another observer’s point of view.

\[ c = \text{car}, \ r = \text{runner}, \ g = \text{ground} \]

\[ (v_x)_c = (v_x)_g + (v_x)_r \]

The speed of the car with respect to the runner is:

[Diagram showing a car moving at 15 m/s and a runner moving at 5 m/s]
Motion on a ramp

An object sliding down a ramp will accelerate parallel to the ramp:

\[ a_x = \pm g \sin \theta \]

The correct sign depends on the direction in which the ramp is tilted.
A car traveling at 23 m/s runs out of gas while traveling up a 9.0 ° slope.

How far will it coast before starting to roll back down?
A ball is thrown horizontally from a 16 m-high building with a speed of 5.0 m/s.

How far from the base of the building does the ball hit the ground?
A skier starts down a $13^\circ$ incline at 5.0 m/s, reaching a speed of 17 m/s at the bottom. Friction between the snow and her freshly waxed skis is negligible.

What is the length of the incline?

How long does it take the skier to reach the bottom?
In 1780, in what is now referred to as "Brady's Leap," Captain Sam Brady of the U.S. Continental Army escaped certain death from his enemies by running over the edge of the cliff above Ohio's Cuyahoga River in (Figure 1), which is confined at that spot to a gorge. He landed safely on the far side of the river. It was reported that he leapt 22 ft ($\approx 6.7$ m) across while falling 20 ft ($\approx 6.1$ m).

What is the minimum speed with which he'd need to run off the edge of the cliff to make it safely to the far side of the river?
A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20 m/s at an angle 5.0 ° above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high.

Does the ball clear the net?

If so, by how much? If not, by how much does it miss?
How do the speeds $v_0$, $v_1$, and $v_2$ (at times $t_0$, $t_1$, and $t_2$) compare?
Consider a diagram of the ball at time $t_0$. (Figure 2) Recall that $t_0$ refers to the instant just after the ball has been launched, so it is still at ground level ($x_0 = y_0 = 0$ m). However, it is already moving with initial velocity $\vec{v}_0$, whose magnitude is $v_0 = 30.0$ m/s and direction is $\theta = 60.0$ degrees counterclockwise from the positive x direction.
What are the values of the initial velocity vector components $v_{0,x}$ and $v_{0,y}$ (both in m/s) as well as the acceleration vector components $a_{0,x}$ and $a_{0,y}$ (both in m/s$^2$)? Here the subscript 0 means "at time $t_0$.”

The peak of the trajectory occurs at time $t_1$. This is the point where the ball reaches its maximum height $y_{\text{max}}$. At the peak the ball switches from moving up to moving down, even as it continues to travel horizontally at a constant rate.

What are the values of the velocity vector components $v_{1,x}$ and $v_{1,y}$ (both in m/s) as well as the acceleration vector components $a_{1,x}$ and $a_{1,y}$ (both in m/s$^2$)? Here the subscript 1 means that these are all at time $t_1$. 
The range $R$ of the ball refers to how far it moves horizontally, from just after it is launched until just before it lands. Range is defined as $x_2 - x_0$, or just $x_2$ in this particular situation since $x_0 = 0$.

Range can be calculated as the product of the flight time $t_2$ and the $x$ component of the velocity $v_x$ (which is the same at all times, so $v_x = v_{0,x}$). The value of $v_x$ can be found from the launch speed $v_0$ and the launch angle $\theta$ using trigonometric functions, as was done in Part B. The flight time is related to the initial $y$ component of the velocity, which may also be found from $v_0$ and $\theta$ using trig functions.

The following equations may be useful in solving projectile motion problems, but these equations apply only to a projectile launched over level ground from position $(x_0 = y_0 = 0)$ at time $t_0 = 0$ with initial speed $v_0$ and launch angle $\theta$ measured from the horizontal. As was the case above, $t_2$ refers to the flight time and $R$ refers to the range of the projectile.
flight time: \( t_2 = \frac{2v_{0,y}}{g} = \frac{2v_0 \sin(\theta)}{g} \)

range: \( R = v_x t_2 = \frac{v_0^2 \sin(2\theta)}{g} \)

In general, a high launch angle yields a long flight time but a small horizontal speed and hence little range. A low launch angle gives a larger horizontal speed, but less flight time in which to accumulate range. The launch angle that achieves the maximum range for projectile motion over level ground is 45 degrees.

Which of the following changes would increase the range of the ball shown in the original figure?
Thank you