Midterm 1
Version A

Last Name: ________________________________

First Name: ________________________________

Student ID: ________________________________

Signature: ________________________________

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Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. You may not use calculators, books, notes, or any other material to help you. Please make sure your phone is silenced and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must show your work to receive credit. Please circle or box your final answers.

Please do not write below this line.

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1. Compute the following limits. If a limit does not exist, be as specific as possible. (E.g. for an infinite limit, find the one-sided limits.) As always, you must justify each answer.

(a) (5 points) \( \lim_{s \to 0} \left( \frac{s + 12}{s^2 + 3s} - \frac{4}{s} \right) \)

(b) (5 points) \( \lim_{t \to 7} \frac{t - 7}{\sqrt{16 - t} - \sqrt{t + 2}} \)
Question 1 continued...

(c) (5 points) \( \lim_{x \to 0} \frac{x + 1}{\sin(x)} \)
2. (12 points) Define a function $f$ as follows, where $a$ and $b$ are unknown constants:

$$f(x) = \begin{cases} \frac{ax+3}{x-5} & \text{if } x \leq 1 \\ \frac{x^3+x^2-2}{x^2-4x+3} & \text{if } 1 < x < 3 \\ x^2 - b & \text{if } x \geq 3 \end{cases}$$

(a) Find the value of the constant $a$ so that $f$ will be continuous at $x = 1$, if this is possible. If not, explain why not.
(b) Find the value of the constant $b$ so that $f$ will be continuous at $x = 3$, if this is possible. If not, explain why not.
3. When a car applies the brakes to come to a stop, its position at time $t$ is given by

$$p(t) = \frac{t^2}{1 + t^2}$$

Use this function to answer the following:

(a) (3 points) Find the average rate of change of the car’s position (average speed) over the interval from $t = 1$ to $t = 3$.

(b) (10 points) Find the instantaneous rate of change of the car’s position (the actual speed) at $t = 1$. *Use the limit definition for this, not just differentiation rules.*

Question 3 continues on the next page...
Question 3 continued...
4. (a) (5 points) Let \( f(x) = 12\sqrt{x} - \frac{2}{x^2} + \frac{1}{\sqrt{x}} - 11 \).

Find the slope of the tangent line to the graph of \( f(x) \) at \( x = 4 \).

(b) (5 points) Let \( g(t) = (t + t^{-1}) \cdot (3t^2 - 5t + 7) \).

Find the instantaneous rate of change of \( g \) at \( t = 1 \).