Math-170B: Probability Theory II
Syllabus

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Topic of the course

Math-170A gave a rigorous, proof-based introduction to the basic objects of mathematical probability, and experience with how they are used to model real-world phenomena. Math-170B is the continuation. It breaks roughly into three parts:

Analysing distributions and their convergence

Some of the most important results of probability theory describe the asymptotic behaviour of certain probability models as some parameter tends to a limit. The law of large numbers and the central limit theorem are both of this kind, describing how the distribution of a sum of \( n \) i.i.d. random variables behaves as \( n \to \infty \). In order to formulate and prove these results, we will look more closely at ways of analysing distributions of random variables, and notions of ‘convergence’ for distributions.

Some basic examples of random processes

Once the foundations have been laid, a lot of work in probability theory takes the following form: some basic quantities are chosen that describe a real-world situation of interest; a probabilistic model is postulated for the random behaviour of those quantities; and then predictions on the typical outcomes in that situation are derived from the model. Several basic examples were already introduced in math-170A. The new examples in math-170B are more advanced: they involve larger collections of random variables, and those random variables depend on either other in more complicated ways. We will introduce and analyze three basic families of such examples: branching processes; random walks on \( \mathbb{Z} \); and Poisson processes. Each of these will require various tools learned previously in math-170AB in order to derive the distribution or asymptotic behaviour of some features or interest.
This part of the course is a precursor to math-171 “Stochastic processes”, which is a more systematic look at this kind of probability theory.

Advanced topics

During the last three weeks, we will meet three more advanced topics in which probability theory plays a role in our understanding of either another part of mathematics or another area of applied science. Details TBA.

Pre-requisites

Math-131A remains an important pre-requisite. Compared to math-170A, in math-170B we will meet some more sophisticated examples of proofs or rigorous derivations in probability theory. These will be a thorough workout for your experience in real analysis and your understanding of logic and operations with sets.

Many concepts from math-170A will reappear during math-170B. I will give the first week to a quick review of math-170A material.

My philosophy regarding homeworks, exams and assessment

I believe that homeworks are of huge value for instruction, but rather little for assessment. The 10% contribution of the homeworks to your overall grade is just a modest incentive to do them. I strongly encourage you to collaborate on homeworks, but the work you submit must be your own.

Many homework questions will be taken from the Grimmett–Welsh book. They are chosen to make you work with and understand the results we are covering in class, and should mostly be of moderate difficulty.

Homeworks will be graded quite strictly, because their main purpose is to give you practice. Grading will be slightly less strict on the exams. The homework grading will take into account clarity as well as correctness: make sure your working is laid out fully and in a logical order. Please don’t panic if your homework grades seem low – it need not mean that you’re doing badly! Feel free to contact me at any time to discuss your progress.

Most exam questions will be a test of competence rather than cleverness. As such, they will be similar to selected homework problems. Homeworks are one of the best ways to prepare for the exams.

The last three weeks of the course are given to a selection of more advanced topics. These may appear on the last one or two homeworks and on the final. If you do well on all problems excluding these topics, but do no questions on these
topics at all, then in principle you can earn a B. But you will need to grapple with these somewhat to earn an A.

Lastly, please be aware that I do not grade to a curve at all. You are not competing with each other for the highest grades.

Feedback

You can give anonymous feedback on any aspect of the course, at any time, using a module on the course CCLE site. Very specific feedback is often the most helpful, even if it cannot be followed to the letter. For instance, feedback which says “The lectures go too fast” is harder to act upon compared with “Give more time to/examples of topic X” or “Go faster on topic X and slower on topic Y”.