Instructor: Michael Willis  
Office: MS 5352  
Class location: MS 7608  
Class meeting time: MWF 10-1050am  
e-mail: msw188@ucla.edu (please include the course number in your subject line)  
Office Hours: TBD  
Course Website: https://ccle.ucla.edu/course/view/20W-MATH236-1  
Textbook: An Introduction to Knot Theory by Lickorish.

Prerequisites: Basic familiarity with topology, especially chain complexes and homology. Knots, links, and cobordisms between them should be defined only up to isotopies, so some comfort with this notion will be essential. For the first half of the course, we may utilize Poincare duality and some covering space theory without proof. We will also use some surgery theory, but will discuss it as needed. We may also mention some connections to representation theory, but no knowledge of this will be assumed.

Course Description: The first half of the course will serve as an introduction to some basic ideas of knot theory in the 3-sphere, including definitions of some polynomial invariants of knots and links. The culmination of the first half of the course will be the use of the (colored) Jones polynomial of a link to give a ‘combinatorial’ definition for the WRT invariant of a 3-manifold (3-dim topology). This part of the course will essentially follow along selected topics from Lickorish, mainly chapters 1-3, 6-9, and 12-13.

The second half of the course will focus on categorifying the Jones polynomial to define a homology theory (Khovanov homology) for knots and links. The main motivation here will be the study of cobordisms between links (4-dim topology). Our main references will be https://arxiv.org/abs/math/0410495 and https://arxiv.org/abs/math/0402131. From here, there are several directions we may take depending on time available and student interest. Some possibilities include:

- colored Khovanov homology, and categorifying Jones-Wenzl projectors via infinite twists;
- a further lift to the stable homotopy category, and an improvement to the s-invariant;
- Khovanov homology in $S^1 \times S^2$, and a curious connection back to certain approaches to attacking the 4-dimensional smooth Poincaré conjecture.

Homework: Homework will be assigned throughout the course, due at the beginning of class on various days. All announcements will be posted on CCLE. You are encouraged to work in groups on the homework, but you must write up your own individual solutions to turn in. Your score on each homework assignment will be based on both accuracy (is the answer correct?) and presentation (is the work/reasoning clearly shown?).
**Exams**: There will be a take-home final exam during finals week. More details will be posted on CCLE closer to the date.

**Course Grade**: Your grade will be determined as follows:

- Homework: 40%
- Exam: 40%
- Class Attendance/Participation: 20%

**Schedule**: Here follows a (very) tentative weekly schedule for the course. The course website will keep a more up-to-date schedule of topics and homework assignments.

- **Week 1 (Jan 6)** - Chapter 1: Links, link diagrams and Reidemeister moves, link complements, linking numbers
- **Week 2 (Jan 13)** - Chapter 2: Seifert surfaces, connected sums, and prime knots
- **Week 3 (Jan 20)** - The Alexander polynomial (some combo of Chapters 6,7,9)
- **Week 4 (Jan 27)** - Skein relations (Ch 8, Ch 3), the Jones polynomial (Ch 3), framed links and Kirby’s theorem for 3-manifolds via surgery (Ch 12)
- **Week 5 (Feb 3)** - Chapter 13: Jones-Wenzl projectors, the colored Jones polynomial, and WRT invariants for 3-manifolds
- **Week 6 (Feb 10)** - Discuss link cobordisms and the desire for a TQFT, Bar-Natan’s approach to Khovanov homology
- **Week 7 (Feb 17)** - More basics with Khovanov homology, and deformations (Lee)
- **Week 8 (Feb 24)** - Rasmussen’s s-invariant and slice genus bounds, reproving Milnor’s conjecture
- **Week 9 (Mar 2)** - More topics
- **Week 10 (Mar 9)** - More topics