Exercise 1 (Convergence in probability). Let $X_1, X_2, \ldots \sim \text{iid } U[-1, 1]$. For each of the following cases, does $Y_1, Y_2, \ldots$ converge in probability to some limit? If so, to what?

1. $Y_n = X_n/n$;
2. $Y_n = X_n^n$;
3. $Y_n = X_1 \cdot X_2 \cdot \ldots \cdot X_n$;
4. $Y_n = \max(X_1, \ldots, X_n)$.

Exercise 2 (Sums converging in probability). Let $X_1, X_2, \ldots$ and $Y_1, Y_2, \ldots$ be sequences of random variables such that $X_n \to X$ and $Y_n \to Y$ both in probability. Prove $X_n + Y_n \to X + Y$ in probability.

Exercise 3 (Notions of convergence). Let $X_n \sim \text{iid Ber}(p)$ (with $p \in (0, 1)$), and let $Y_n = \max\{X_1, \ldots, X_n\}$. Does $Y_n$ converge

(a) in probability?
(b) almost surely?
(c) in distribution?
(d) in $L^1$?

Exercise 4 (Continuous waiting time). Fix $\lambda > 0$, and let $X_n \sim \text{Geom}(\lambda n)$ (where $n$ is chosen to be sufficiently large so that $\lambda n < 1$). Prove that $\frac{X_n}{n}$ converges in distribution to $X \sim \text{Exp}(\lambda)$.