HOMEWORK ASSIGNMENTS

Section 1.1: #1.1.5 (assume the Cauchy-Schwarz inequality), #1.1.6, #1.1.16

Section 1.2: Let \( E = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, y > 0\} \). Determine, without proofs, whether the following points are interior, exterior or boundary points of \( E \): \((0, 0), (-1/2, 1/4), (1/\sqrt{2}, 1/\sqrt{2}), (1, 1), (1, 0), #1.2.2, #1.2.3(c), #1.2.3(f)\)

Section 1.3: #1.3.1

Section 1.4: #1.4.1, #1.4.3, #1.4.4

Section 1.5: #1.5.6, #1.5.7(b), #1.5.7(c), #1.5.9, #1.5.11, #1.5.14, #1.5.15

Section 2.1: #2.1.1((c) implies (b)), #2.1.2((c) if and only if (d)), #2.1.3

Section 2.2: #2.2.1(\( f \oplus g \) continuous, so are \( f \) and \( g \)), #2.2.2(\( f, g \) continuous, so is \( f - g \)), #2.2.2(\( f \) continuous at \( x_0 \) and \( c \in \mathbb{R} \), then \( cf \) continuous at \( x_0 \)), #2.2.2(\( f, g \) continuous, so is \( \min(f, g) \)), #2.2.2(\( f, g \) continuous at \( x_0 \) and \( g(x_0) \neq 0 \) then \( f/g \) continuous at \( x_0 \))

Section 2.3: #2.3.1, #2.3.4, #2.3.5, #2.3.6(\( f, g \) uniformly continuous, so is \( f + g \)), #2.3.6(\( M(x, y) = xy \) is not uniformly continuous)

Section 2.4: #2.4.4, #2.4.5, #2.4.6, #2.4.7, #2.4.8

Section 3.1: #3.1.2(a), #3.1.2(b), #3.1.2(c)

Section 3.2: Prove that if \( f_n : X \rightarrow Y \) converges uniformly to \( f \) and \( g : Y \rightarrow Z \) is uniformly continuous, then \( g \circ f_n : X \rightarrow Z \) converges uniformly to \( g \circ f \), #3.2.4(\( Y = \mathbb{R}, y_0 = 0 \)).

Section 3.3: #3.3.4, #3.3.6, Prove that if \( f^{(n)} : X \rightarrow Y \) converge
uniformly to \( f \) and the \( f^{(n)} \) are uniformly continuous, then \( f \) is also uniformly continuous.

Section 3.4: #3.4.1, #3.4.2

Section 3.5: (1) Prove that if a sequence \( f^{(n)}: X \to \mathbb{R} \) is uniformly convergent, then it is uniformly Cauchy. (2) Prove that if \( \sum f^{(n)} \) and \( \sum g^{(n)} \) converge uniformly, so also does \( \sum (f^{(n)} + g^{(n)}) \). (3) Prove that if \( \sum f^{(n)} \) converges uniformly, then \( \lim_{n \to \infty} \|f^{(n)}\| = 0 \).

Section 3.6: #3.6.1

Section 3.7: #3.7.3

Section 4.1: Prove the Root Test, #4.1.1(a), #4.1.1(b), #4.1.1(d)

Section 4.2: #4.2.2, #4.2.3, #4.2.4

Section 4.5: #4.5.1(e)(f), #4.5.3, #4.5.5(b)(c)(d)(e) (assume \( (e^x)^y = e^{xy} \))

Section 6.2: #6.2.1, #6.2.2

Section 6.3: #6.3.1, #6.3.2